# Summation of the Diagrams in the HBChPT: New Results for the Magnetic Moments of the Baryon Octet

**S.B. Yakovlev, V.S. Zamiralov**

NPI MSU Preprint 2003–11/724

MOSCOW 2003

M.V. Lomonosov Moscow State University

D.V. Skobeltsyn Institute of Nuclear Physics

# Summation of the Diagrams in the HBChPT: New Results for the Magnetic Moments of the Baryon Octet [1]

**S.B. Yakovlev, V.S. Zamiralov**

NPI MSU Preprint 2003–11/724

MOSCOW 2003

[1]Report on the X International Seminar
"Electromagnetic Interactions of Nuclei at Low and Medium Energies",
April 16-18, 2003, Moscow, Russia.

УДК 539.173

S.B. Yakovlev, V.S. Zamiralov

D.V. Skobeltsyn Institute of Nuclear Physics, Moscow State University, Moscow, Russia
e-mail: zamir@depni.npi.msu.su
preprint NPI MSU 2003-11/724

# Summation of the diagrams in the HBChPT: New Results for the Magnetic Moments of the Baryon Octet


## Summary

Magnetic moments of spin 1/2 octet baryons are reevaluated in framework of Heavy Baryon Chiral Perturbation Theory (HB$\chi$PT). New formulae for the tree-level approximation are proposed. Calculations of one-loop contributions to magnetic moments are performed in terms of the $SU(3)_f$ couplings of octet baryons to Goldstone bosons. It is shown that contributions of the counter terms needed in the fit prove to be large and comparable to the one-loop contributions. It is shown that upon using natural units for the baryon magnetic moments it is possible to somewhat improve the results. Still we see that one-loop corrections without counter terms do not change the main results of the $SU(3)$ symmetry scheme which gives similar fit upon using simple assumptions and only two free parametrs.



## Резюме

Магнитные моменты барионов октета спина 1/2 проанализированы в рамках киральной теории возмущений для тяжелых барионов (HHCPT). Предложены новые формулы для магнитных моментов в древесном приближении. Вычисления однопетлевых поправок проведены в терминах констант связи в модели $SU(3)_f$. Показано, что вклады контрчленов, необходимых при фите, велики и сравнимы с однопетлевыми вкладами. Показано, что использование естественных единиц для магнитных моментов барионов несколько улучшает результаты. Однако видно, что однопетлевые поправки без контрчленов не изменяют существенно результатов унитарной модели, которая дает близкий фит, используя простые предположения и только два свободных параметра.

# 1 Introduction

Recently magnetic moments of the octet and decuplet baryons as well as of charm baryons were analyzed thoroughly in the framework of the Heavy Baryon Chiral Perturbation Theory (HBChPT) [1-6]. It is a hope that on this way the convergent chiral perturbation theory series for magnetic moments of the octet baryons could be constructed. The tree-level values of the baryon magnetic moments are given either by the $SU(3)_f$ model [8] or by the nonrelativistic quark model (QM) [9] with/or without some modifications. Indeed, already QM describes baryon magnetic moments up to $0.12\mu_N$ ($\mu_N$ means nuclear magneton). The deviations left present a sensitive test of the baryon structure and are treated in the framework of the HB$\chi$PT model through one-loop corrections and counter terms.

Within this scheme, upon using known values of the axial-vector constants $F$ and $D$ and usual assumptions as to various decuplet couplings, one is left with a few free parameters to adjust seven known octet baryon magnetic moments and magnetic moment of the $\Omega$-hyperon.

We follow the main trend of the works [1-6] and show that a sum of all one-loop contributions reduces to a very symmetrical form. Moreover, with the degenerated masses of $\pi$ and $K$ -mesons ( which play a role of Goldstone bosons ) all the chiral contributions can be effectively eliminated through redefinition of the quantities $\mu_F, \mu_D$'s of the old-fashioned $SU(3)_f$ model [8].

For the octet and decuplet baryons the results of the HB$\chi$PT model seems to show nice convergent pattern[3, 6]. But this pattern requires large counter-term contributions in order to fit the data. We write explicit formulae to show it more clear. We propose here to use natural magnetic units for baryons which serves to arrive at smaller counter-term contributions.

Nevertheless the results of HB$\chi$PT still are at the accuracy of a simple-minded $SU(3)$ model. We hope that some more appropriate treatment of the mass-breaking terms and/or using of natural units for the baryon magnetic moments would allow to arrive at more convergent scheme.

The report is organized as follows. In the 2nd section there are elements of the Heavy Baryon Chiral Perturbation Theory (HB$\chi$PT). New formulae for magnetic moments of the octet baryons are given. In the 3rd section we briefly repeat calculation of the unitary symmetry part of the one-loop contributions to the magnetic moments and present it in a form suitable for group-theoretical treatment of the problem. Many results are given explicitly.

The conclusions are stated at the end of the report.

# 2 Chiral Lagrangians of the HB$\chi$PT

We expose necessary elements of the Heavy Baryon Chiral Perturbation Theory (HB$\chi$PT), basing mainly on the works [1, 3, 6, 10, 11]. In its framework a chiral expansion of the baryon Lagrangian is written in terms of the velocity-dependent octet and decuplet fields $B_v(x)$, $T_v(x)$ constructed in order to remove the free momentum dependence in the Dirac equation,

$$B_v(x) = exp(iM_B\hat{v}v{\cdot}x)B(x); \quad T^\mu_v(x) = exp(iM_B\hat{v}v{\cdot}x)T^\mu(x).$$

Here $B(x)$ and $T^{\mu}(x)$ are the baryon octet 1/2 and decuplet 3/2 fields with the masses $M_B$ and $M_T$ , respectively. Note that in the definition of the $T^{\mu}_v(x)$ stays the mass $M_B$. We define also the difference $\delta = M_T - M_B$ and use in calculations $\delta = 250MeV$. The $B_v(x)$ octet is given by the matrix

$$B_v = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0_v + \frac{1}{\sqrt{6}}\Lambda^0_v & \Sigma^+_v & p_v \\ \Sigma^-_v & -\frac{1}{\sqrt{2}}\Sigma^0_v + \frac{1}{\sqrt{6}}\Lambda^0_v & n_v \\ \Xi^-_v & \Xi^0_v & -\frac{2}{\sqrt{6}}\Lambda^0_v \end{pmatrix}. \quad (1)$$

The $T_v(x)$ is given by the symmetric tensor of the 3rd rank in the unitary space, $T_v(x) \equiv T^{ijk}_v(x), \quad i,j,k = 1,2,3$ being $SU(3)_f$ indices ($T^{111}_v(x) = \Delta^{++}(x)$ etc.). The Goldstone bosons appearing in the limit of chiral symmetry are identified with the pseudoscalar octet and are parametrized as follows

$$P = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}. \quad (2)$$

This pseudoscalar octet couples to the baryon fields via the vector and axial vector currents

$$V^{\mu} = \frac{1}{2}(\xi\partial^{\mu}\xi^{\dagger} + \xi^{\dagger}\partial^{\mu}\xi), \quad A^{\mu} = \frac{i}{2}(\xi\partial^{\mu}\xi^{\dagger} - \xi^{\dagger}\partial^{\mu}\xi),$$

where $\xi = exp(iP/f)$ and $\xi \to L\xi R^{\dagger}$, with $L, R \in SU(3)_{L,R}$ and $f = f_{\pi} \approx 93MeV$ being the pseudoscalar decay constant in the chiral limit. In the fit $f_{\pi} = 93$MeV, $f_K = f_{\eta} = 1.2f_{\pi}$ were used [3].

The lowest order chiral Lagrangian for octet and decuplet baryons reads [1]

$$L^0_v = iSp\bar{B}_v(v\mathcal{D})B_v + 2DSp\bar{B}_vS^{\mu}_v\{A_{\mu}, B_v\} + 2FSp\bar{B}_vS^{\mu}_v[A_{\mu}, B_v] - \\ i\bar{T}^{\mu}_v(v\mathcal{D})T_{v\mu} + \delta\bar{T}^{\mu}_vT_{v\mu} + C(\bar{T}^{\mu}_vA_{\mu}B_v + \bar{B}_vA_{\mu}T^{\mu}_v) + ... \quad (3)$$

where $\mathcal{D}_{\mu} = \partial_{\mu} + [V_{\mu},]$ is the covariant chiral derivative.and $F$ are characteristic for $SU(3)_f$ octet baryon coupling constants with the axial-vector field, while $C$ is a relevant coupling constant for the decuplet-octet axial-vector transition . Taking $F = 2/3D, C = -2D, D = 1$ one returns to NRQM. In the calculations $D = 0.75$, $F = 0.5, C = -1.5$ were used. The operator $S^{\mu}_v$ is defined in [2, 10] and reduces to $S^{\mu}_v = (0, \frac{1}{2}\sigma)$ in the frame in which $v^{\mu} = (1,0,0,0)$.

The electromagnetic interaction is introduced via $\mathcal{D}_{\mu} \to \mathcal{D}_{\mu} + ie\mathcal{A}^{el}_{\mu}[Q,]$ and $\partial_{\mu}P \to \mathcal{D}_{\mu}P$, where $\mathcal{A}^{el}_{\mu}$ is the photon field and $Q$ is the charge matrix for the quarks $u, d$ and $s$, $Q = diag(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3})$.

As it was said, one begins calculation of the magnetic moments with the tree-level approximation and then performs one-loop corrections to it. There are also one-loop diagrams where one inserts tree-level magnetic moments of the octet and decuplet baryons and the corresponding transition magnetic moments. So we need tree-level magnetic moments.

For the octet baryons tree-level magnetic moments with the counter-terms due to mass-breaking can be written in the form [1, 3]

$$L_{BS} = \frac{e}{4M_N}\Big\{\mu_D Sp(\bar{B}_vF_{\mu\nu}\sigma^{\mu\nu}\{Q, B_v\}) + \mu_F Sp(\bar{B}_vF_{\mu\nu}\sigma^{\mu\nu}[Q, B_v]) + \\ F_{\mu\nu}[Sp(b_3\bar{B}_v\sigma^{\mu\nu}[[Q, B_v], \mathcal{M}]) + b_4Sp(\bar{B}_vQ\sigma^{\mu\nu}\{[Q, B_v]\mathcal{M}\}) + \\ b_5Sp(\bar{B}_v\sigma^{\mu\nu}[\{Q, B_v\}, \mathcal{M}]) + b_6Sp(\bar{B}_v\sigma^{\mu\nu}\{\{Q, B_v\}\mathcal{M}\}) + b_7Sp(\bar{B}_v\sigma^{\mu\nu}B_v)Sp(\mathcal{M}Q))]\Big\}, \quad (4)$$

where $b's$ are counterterm coupling constants and $\mathcal{M} = diag(0,0,1)$ introduces mass-breaking term. As it was shown in [13, 14], this expression can be casted in the form containing unitary symmetry magnetic moments [8], symmetry-breaking terms due to mass corrections and effective meson-cloud contributions. Here we would prefer to state them in another way. A tree-level magnetic moment of any hyperon ( without counter terms) with the $\Sigma(qq,h)$-like quark wave function ( that is for all of them but $\Lambda$ hyperon) containing two like quarks $q$ with the charge $e_q$ and a single quark $h$ with the charge $e_h$ would have the form

$$\mu(\Sigma(qq,h)) = 2e_q\mu_F + e_h(\mu_F - \mu_D). \tag{5}$$

One reproduces immediately the Coleman-Glashow relations [8].

$$\mu(p) = \mu(\Sigma^+) = \mu_F + \frac{1}{3}\mu_D,$$

$$\mu(\Sigma^-) = \mu(\Xi^-) = -\mu_F + \frac{1}{3}\mu_D,$$

$$\mu(n) = \mu(\Xi^0) = -\frac{2}{3}\mu_D.$$

Similarly for the $\Sigma$-like hyperon with two different quark in the biquark state, $\Sigma(qq',h)$, one can write

$$\mu(\Sigma(qq',h)) = (e_q + e_{q'})\mu_F + e_h(\mu_F - \mu_D). \tag{6}$$

One can write also a formula for the magnetic moments of $\Lambda$ hyperon containing two light quarks $u, d$ and one heavy quark $s$ as

$$\mu_{\Lambda(uds)} = (e_u + e_d)(\mu_F - \frac{2}{3}\mu_D) + e_s(\mu_F + \frac{1}{3}\mu_D). \tag{7}$$

One obtains the known $SU(3)$ result, namely, $\mu_\Lambda = -\mu_D/3$ [8].

The counter terms can be included into the formula as

$$\mu_{\Sigma(qq,h)} = 2e_q\mu_F + e_h(\mu_F - \mu_D) + T + \frac{1}{2}\alpha(qq,h),$$

$$\alpha(uu,d) + \alpha(uu,s) = 0, \qquad \alpha(ss,d) + \alpha(dd,s) = 0,$$

$$\alpha(ss,u) + \alpha(dd,u) = 0.$$

$$\begin{pmatrix} \mu_D \\ \mu_F \\ T \\ \alpha(uu,d) \\ \alpha(dd,s) \\ \alpha(ss,u) \\ l_\lambda \end{pmatrix} = \begin{pmatrix} 1 & 0 & \frac{1}{2} & 0 & 0 & \frac{5}{6} & 0 \\ 0 & 1 & 0 & \frac{1}{2} & 0 & \frac{1}{6} & 0 \\ 0 & 0 & \frac{1}{3} & 0 & 0 & -\frac{1}{9} & -\frac{1}{3} \\ 0 & 0 & 1 & 1 & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 0 & -1 & 1 & \frac{1}{3} & -\frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & \frac{4}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{3} & -\frac{5}{9} & -\frac{1}{6} \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \\ b_7 \end{pmatrix} \tag{8}$$

In [12] these 7 constants were used to describe magnetic moments of the octet baryons, provided only two of them , namely $b_1$ and $b_2$, are large. Instead recently in [6], where calculations were performed up to $O(1/\Lambda_\chi^3)$, the parameters $b_k's$ ($k = 3, ...7$) which intrinsically are of the order $O(1/\Lambda_\chi^3)$ were adjusted by a fit to give tree-level values of the baryon magnetic moments.

The decuplet-octet transition magnetic operator can be taken as in [6]

$$L^{(10-8)} = i\frac{\tilde{\mu}_T}{\Lambda_\xi} F_{\mu\nu}(\epsilon_{ijk}\hat{Q}^i_l \bar{B}^j_{vm} S^\mu_v T^{\nu klm}_v + H.C.),$$

where $i, j, k, l, m$ are $SU(3)_f$ indices and $\hat{Q} = diag(2/3, -1/3, -1/3)$. The value $\tilde{\mu}_T = -4.79 \pm 0.31$ taken from the measured value of the $\Delta \to \gamma N$ decay is just the value used in [1] $\mu_T = -7.7 \pm 0.5$, as $\tilde{\mu}_T/\Lambda_\xi = \mu_T/2m_N$. With $\hat{Q} = diag(\mu_u, \mu_d, \mu_s)$ and $\tilde{\mu}_T/\Lambda_\xi$ changed to $e/2m_N$ this Lagrangian gives the decuplet-octet transition magnetic moments of the usual NRQM.

Now we proceed calculations of the one-loop corrections to the octet baryon magnetic moments in the H$\chi$ChPT along the lines of [1] and [6].

# 3 The one-loop corrections to the octet baryon magnetic moments

The octet baryon magnetic moments in the H$\chi$ChPT includes tree-level contributions (with the corresponding counter terms) and non-analytic corrections arising from one-loop diagrams, involving $\pi$, $K$ and $\eta$ loops with octet and decuplet baryon insertions. We choose to write it in a simple form proposed in [1]

$$\mu_B = \mu_B^0 + \left\{ \sum_{X=\pi,K} \frac{m_N}{8\pi f^2}[M_X\beta_B^{(X)} + F(M_X,\delta,\mu)\tilde{\beta}_B^{(X)}] + \right. \\ \sum_{X=\pi,K,\eta} \frac{1}{32\pi^2 f^2}(\bar{\gamma}_B^{(X)} - 2\bar{\lambda}_B^X \mu_B^0) M_X^2 ln(M_X^2/\mu^2) \tag{9}$$

where $\mu_B^0$ are tree-level magnetic moments. The $\bar{\lambda}_B^{\pi,K,\eta} = \lambda_{B,8}^{\pi,K,\eta} + \lambda_{B,10}^{\pi,K,\eta}$ are the sum of pion-, kaon- and $\eta$-loop self-energy contributions with the octet and decuplet baryons in the loop. The $\beta_B^{\pi,K}$ and $\tilde{\beta}_B^{\pi,K}$ are the contributions from the $\pi$ and $K$ loops with the photon line attached to the meson with intermediate octet and decuplet states, respectively [1] . The $\gamma_B^{(\pi,K,\eta)}$'s are sums of the contributions from the $\pi$, $K$ and $\eta$ loops with the photon attached to the 8, 10 and 8-10 baryon insertions. The renormalization scale $\mu$ was taken here equal to 1 GeV. We take degenerate $K$ and $\eta$ mesons. The loop integral $F(M_X, \delta, \mu)$ is [1]:

$$\pi F(m,\delta,\mu) = -\delta ln\frac{m^2}{\mu^2} + 2\sqrt{m^2-\delta^2}(\frac{\pi}{2} - arctan[\frac{\delta}{\sqrt{m^2-\delta^2}}]), \quad \delta \le m$$

$$\delta ln\frac{m^2}{\mu^2} + \sqrt{m^2-\delta^2} ln\frac{\delta - \sqrt{m^2-\delta^2}}{\delta + \sqrt{m^2-\delta^2}}, \quad \delta > m$$

We prefer for $\Sigma$-like baryons to rewrite Eq.(9) in the form

$$\mu_{\Sigma(qq,h)} = 2c_q\mu_F^{corr} + c_h(\mu_F^{corr} - \mu_D^{corr}) + T^{corr} + \frac{1}{2}\alpha(qq,h)^{corr}, \tag{10}$$

where

$$\mu_F^{corr} = \mu_F + F^{sf} + F^{mes} + F^{bar}, \quad F^{sf} = F^{sf8} + F^{sf10} \quad etc,$$

and similarly for $\mu_D^{corr}$, $T^{corr}$, $\alpha(qq,h)^{corr}$'s.

### 3.1. One-loop self-energy contributions to the baryon magnetic moments

We have calculated self-energy graphs of the type shown in Fig.1. It is convenient to write the result in the form used for the tree-level magnetic moments

$$\mu^{sf}(B(qq,q')) = \mu_0^{sf}(B) + 2c_q F^{sf} + c_{q'}(F^{sf} - D^{sf}) + T^{sf} + \frac{1}{2}\alpha^{sf}(qq,q'),$$

where the last term is proportional to the value of the $U$-spin projection of the baryon and

$$\alpha^{sf}(uu,d) = -\alpha^{sf}(uu,s) \equiv \alpha^{sf}_{p\Sigma^+}, \qquad \alpha^{sf}(ss,d) = -\alpha^{sf}(dd,s) \equiv \alpha^{sf}_{\Xi^-\Sigma^-},$$

$$\alpha^{sf}(ss,u) = -\alpha^{sf}(dd,u) \equiv \alpha^{sf}_{\Xi^0 n}.$$

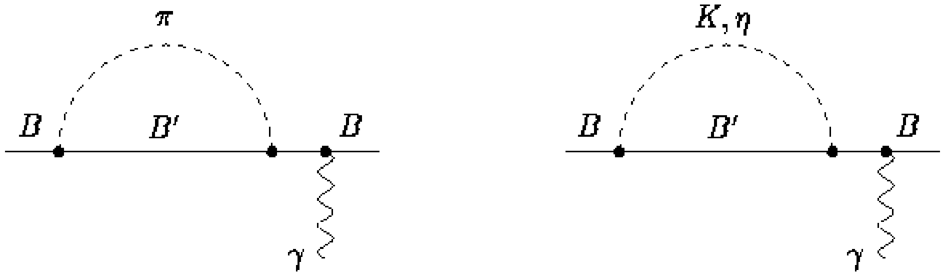



Fig.1

For baryons $B(qq,q')$ of the octet it yields

$$\mu^{sf}(p) = \mu_0^{sf}(p) + F^{sf} + \frac{1}{3}D^{sf} + T^{sf} + \frac{1}{2}\alpha^{sf}_{p\Sigma^+},$$

$$\mu^{sf}(\Sigma^+) = \mu_0^{sf}(\Sigma^+) + F^{sf} + \frac{1}{3}D^{sf} + T^{sf} - \frac{1}{2}\alpha^{sf}_{p\Sigma^+}$$

$$\mu^{sf}(\Xi^-) = \mu_0^{sf}(\Xi^-) - F^{sf} + \frac{1}{3}D^{sf} + T^{sf} + \frac{1}{2}\alpha^{sf}_{\Xi^-\Sigma^-}$$

$$\mu^{sf}(\Sigma^-) = \mu_0^{sf}(\Sigma^-) - F^{sf} + \frac{1}{3}D^{sf} + T^{sf} - \frac{1}{2}\alpha^{sf}_{\Xi^-\Sigma^-}$$

$$\mu^{sf}(\Xi^0) = \mu_0^{sf}(\Xi^0) - \frac{2}{3}D^{sf} + T^{sf} + \frac{1}{2}\alpha^{sf}_{\Xi^0 n}$$

$$\mu^{sf}(n) = \mu_0^{sf}(n) - \frac{2}{3}D^{sf} + T^{sf} - \frac{1}{2}\alpha^{sf}_{\Xi^0 n}$$

$$\mu^{sf}(\Lambda) = \mu_0^{sf}(\Lambda) - \frac{1}{3}D^{sf} + L^{sf}_\Lambda + T^{sf}$$

$$\sqrt{3}\mu^{sf}(\Sigma^0\Lambda) = \sqrt{3}\mu_0^{sf}(\Sigma^0\Lambda) - D^{sf} + SL^{sf}_{\Sigma^0\Lambda},$$

where

$$\mu_0^{sf}(B) = \frac{1}{2}\mu_0(B)[(9F^2 + 5D^2)I^{sf8}_{(\pi+K)} + \frac{5}{2}C^2 I^{sf10}_{(\pi+K)};$$

$$T^{sf} = \frac{1}{3}[A^{sf}_{p\Sigma^+} + B^{sf}_{\Xi^-\Sigma^-} + C^{sf}_{\Xi^0 n}],$$

$$D^{sf} = \frac{1}{2}[A^{sf}_{p\Sigma^+} + B^{sf}_{\Xi^-\Sigma^-} - 2C^{sf}_{\Xi^0 n}], \quad F^{sf} = \frac{1}{2}[A^{sf}_{p\Sigma^+} - B^{sf}_{\Xi^-\Sigma^-}],$$

$$A^{sf}_{p\Sigma^+} = \frac{1}{8}(\mu_F + \frac{1}{3}\mu_D)[(15F^2 - 5D^2 - 18FD)I^{sf8}_{(\pi-K)} - \frac{2}{3}C^2 I^{sf10}_{(\pi-K)};$$

$$B^{sf}_{\Xi^-\Sigma^-} = \frac{1}{8}(-\mu_F + \frac{1}{3}\mu_D)[(15F^2 - 5D^2 + 18FD)I^{sf8}_{(\pi-K)} - \frac{20}{3}C^2 I^{sf10}_{(\pi-K)};$$

$$C^{sf}_{\Xi^0 n} = \frac{1}{4}(9F^2 + D^2)(-\frac{2}{3}\mu_D)I^{sf8}_{(\pi-K)}$$

$$L^{sf}_{\Lambda} = \frac{1}{2}(-\frac{1}{3}\mu_D)[(-9F^2 + D^2)I^{sf8}_{(\pi-K)} + C^2 I^{sf10}_{(\pi-K)};$$

$$\alpha^{sf}_{p\Sigma^+} = \frac{1}{4}(\mu_F + \frac{1}{3}\mu_D)[(3F^2 + 7D^2 - 18FD)I^{sf8}_{(\pi-K)} + \frac{5}{3}C^2 I^{sf10}_{(\pi-K)};$$

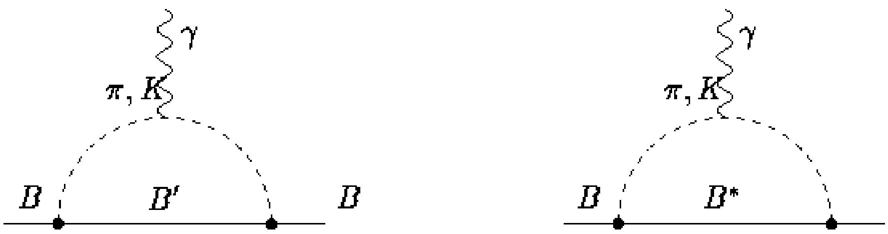
$$\alpha^{sf}_{\Xi^-\Sigma^-} = \frac{1}{4}(-\mu_F + \frac{1}{3}\mu_D)[(3F^2 + 7D^2 + 18FD)I^{sf8}_{(\pi-K)} + \frac{1}{6}C^2 I^{sf10}_{(\pi-K)};$$

$$\alpha^{sf}_{\Xi^0 n} = \mu_D[-6FDI^{sf8}_{(\pi-K)} + C^2 I^{sf10}_{(\pi-K)}];$$

Here $I^{sf8}_{(\pi,K)} = M_{\pi,K}m_N/8\pi f^2_{\pi,K}$, $I^{sf10}_{(\pi,K)} = F(M_{\pi,K},\delta,\mu)m_N/8\pi f^2_{\pi,K}$, $I^{sf8,10}_{(\pi\pm K)} = I^{sf8,10}_{(\pi)} \pm I^{sf8,10}_{(K)}$. Note that with the masses of $\pi, \eta$ and $K$ mesons degenerated and $f_\pi = f_\eta = f_K$ we return to $SU(3)$ model with just redefined $\mu_F, \mu_D$.

**3.2. One-loop meson current contributions**

Now we calculate one-loop corrections due to meson currents with octet and decuplet baryons in the loops given by the diagrams:




.  Fig.2

The octet and decuplet contributions can also be put in a form similar to that of the tree-level magnetic moments:

$$\mu^{mes8,10}(B(qq,q')) = 2c_q F^{mes8,10} + c_{q'}(F^{mes8,10} - D^{mes8,10}) + \frac{1}{2}\alpha^{mes8,10}(qq,q'),$$

**Meson currents with the octet insertions in the loop**

For the loops with the octet contributions one get

$$D^{mes8} = -3DFI^{mes8}_{(\pi+K)}, \qquad F^{mes8} = -2(\frac{5}{3}D^2 + 3F^2)I^{mes8}_{(\pi+K)}, \quad T^{mes8} = 0,$$

$$\alpha^{mes8}_{p\Sigma^+} = -(\frac{1}{3}D^2 - 3F^2 + 2FD)I^{mes8}_{(\pi-K)}, \qquad \alpha^{mes8}_{\Xi^-\Sigma^-} = (\frac{1}{3}D^2 - 3F^2 - 2FD)I^{mes8}_{(\pi-K)},$$

$$\alpha^{mes8}_{\Xi^0 n} = -2(F^2 + D^2)I^{mes8}_{(\pi-K)}. \qquad L^{mes8}_{\Lambda} = \sqrt{3}SL^{mes8}_{\Sigma^0\Lambda} = -FDI^{mes8}_{(\pi-K)}.$$

**Meson currents with the decuplet insertions in the loop**

Now we go to calculation of the loop corrections with the decuplet insertions:

$$D^{mes10} = -\frac{1}{4}C^2 I^{mes10}_{(\pi+K)}, \quad F^{mes10} = T^{mes10} = 0, \qquad L^{mes10}_{\Lambda} = \sqrt{3}SL^{mes10}_{\Sigma^0\Lambda} = -\frac{1}{12}C^2 I^{mes10}_{(\pi-K)},$$

$$\alpha^{mes10}_{p\Sigma^+} = -\frac{5}{18}C^2 I^{mes10}_{(\pi-K)}, \quad \alpha^{mes10}_{\Xi^0 n} = 2\alpha^{mes10}_{\Xi^-\Sigma^-} = -\frac{1}{9}C^2 I^{mes10}_{(\pi-K)}.$$

Note that with the masses of $\pi$ and $K$ mesons degenerated we return to $SU(3)$ model with redefined $\mu_F, \mu_D$.

**3.3. One-loop baryon current contributions**

Finally we calculate baryon current contributions given by the diagrams:

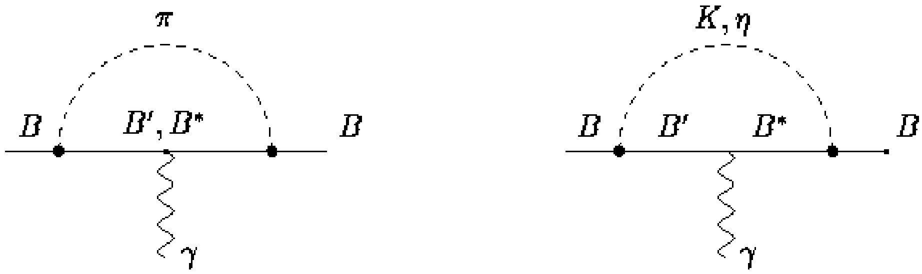



Fig.3

and begin from octet baryon insertions.

$$\hat{D}^{bar8} = \frac{1}{2}[(D^2 - 3F^2)\mu_D + 6FD\mu_F]I^{bar8}_{(\pi+K)} + [(-\frac{5}{18}D^2 + \frac{13}{6}F^2)\mu_D - 3FD\mu_F]I^{bar8}_{(\pi-K)},$$

$$\hat{F}^{bar8} = [\frac{5}{3}FD\mu_D - (\frac{5}{6}D^2 + \frac{3}{2}F^2)\mu_F]I^{bar8}_{(\pi+K)} + [-\frac{1}{2}FD\mu_D + (\frac{7}{18}D^2 + \frac{1}{4}F^2)\mu_F]I^{bar8}_{(\pi-K)},$$

$$T^{bar8} = \frac{1}{3}[(\frac{13}{18}D^2 - \frac{5}{6}F^2)\mu_D - 3FD\mu_F]I^{bar8}_{(\pi-K)},$$

$$L^{bar8}_{\Lambda} = [(-\frac{85}{108}D^2 + \frac{5}{36}F^2)\mu_D + \frac{3}{2}DF)\mu_F]I^{bar8}_{(\pi-K)},$$

$$\sqrt{3}SL^{bar8}_{\Sigma^0\Lambda} = [(-\frac{37}{36}D^2 + \frac{5}{12}F^2)\mu_D + \frac{5}{2}DF)\mu_F]I^{bar8}_{(\pi-K)},$$

$$\alpha^{bar8}_{p\Sigma^+} = \frac{1}{2}[(\frac{5}{9}D^2 + \frac{11}{3}F^2 - \frac{2}{3}FD)\mu_D + (-D^2 + 3F^2 - 2DF)\mu_F]I^{bar8}_{(\pi-K)},$$

$$\alpha^{bar8}_{\Xi^-\Sigma^-} = \frac{1}{2}[(\frac{5}{9}D^2 + \frac{11}{3}F^2 + \frac{2}{3}FD)\mu_D - (-D^2 + 3F^2 + 2DF)\mu_F]I^{bar8}_{(\pi-K)},$$

$$\alpha^{bar8}_{\Xi^0 n} = 2(D^2 + F^2)\mu_F I^{bar8}_{(\pi-K)}.$$

**Baryon currents with 10-8 insertions in the loop**

Now we treat mixed 8-10 insertions into the loops:

$$\hat{D}^{bar10-8} = [(\frac{2}{3}D + \frac{8}{9}F)I^{bar10-8}_{\pi} + (\frac{2}{3}D + \frac{10}{9}F)I^{bar10-8}_{K}]C\mu_T,$$

$$\hat{F}^{bar10-8} = [(\frac{4}{9}D + \frac{2}{9}F)I^{bar10-8}_{\pi} + (\frac{2}{3}D - \frac{2}{9}F)I^{bar10-8}_{K}]C\mu_T,$$

$$T^{bar10-8}=\frac{2}{3}(-\frac{1}{3}D+\frac{2}{9}F)\mathcal{C}\mu_T I^{bar10-8}_{(\pi-K)},\quad L^{bar10-8}_{\Lambda}=(-\frac{1}{3}D+\frac{1}{27}F)\mathcal{C}\mu_T I^{bar10-8}_{(\pi-K)},$$

$$\sqrt{3}SL^{bar10-8}_{\Sigma^0\Lambda}=-(\frac{5}{9}D+\frac{2}{9}F)\mathcal{C}\mu_T I^{bar10-8}_{(\pi-K)},$$

$$\alpha^{bar10-8}_{p\Sigma^+}=(\frac{2}{3}D+\frac{2}{9}F)\mathcal{C}\mu_T I^{bar10-8}_{(\pi-K)},\quad \alpha^{bar10-8}_{\Xi^-\Sigma^-}=\frac{2}{9}(D-F)\mathcal{C}\mu_T I^{bar10-8}_{(\pi-K)},$$

$$\alpha^{bar10-8}_{\Xi^0 n}=(\frac{2}{3}D+\frac{10}{9}F)\mathcal{C}\mu_T I^{bar10-8}_{(\pi-K)}$$

**Baryon currents with decuplet baryons in the loop**

Finally we consider insertion of the baryon decuplet

$$\hat{D}^{bar10}=-\frac{1}{6}[3I^{bar10}_{\pi}+I^{bar10}_{K}]\mathcal{C}^2\mu_C,\quad \hat{F}^{bar10}=-\frac{1}{54}[19I^{bar10}_{\pi}+41I^{bar10}_{K}]\mathcal{C}^2\mu_C,$$

$$T^{bar10}=-\frac{1}{9}\mathcal{C}^2\mu_C I^{bar10}_{(\pi-K)},\quad L^{bar10}_{\Lambda}=-\sqrt{3}SL^{bar10}_{\Sigma^0\Lambda}=-\frac{1}{18}\mathcal{C}^2\mu_C I^{bar10}_{(\pi-K)},$$

$$\alpha^{bar10}_{p\Sigma^+}=-\frac{10}{9}\mathcal{C}^2\mu_C I^{bar10}_{(\pi-K)},\quad \alpha^{bar10}_{\Xi^-\Sigma^-}=0,\quad \alpha^{bar10}_{\Xi^0 n}=-\frac{4}{27}\mathcal{C}^2\mu_C I^{bar10}_{(\pi-K)}.$$

Note that with the masses of $\pi,\eta$ and $K$ mesons degenerated and $f_\pi=f_\eta=f_K$ we return to $SU(3)$ model with redefined $\mu_F,\mu_D$.

# 4 Results and discussion

First we display results for the octet contributions into the loops up to $1/\Lambda^3_\chi$ order in chiral expansion basing on the reasoning of [6]. The first term of each line is the $1/\Lambda_\chi$ order term (with the coefficient 1 in the brackets), the 2nd term is the meson current contribution (cf diagrams of the Fig.2) of the $1/\Lambda^2_\chi$ order. The last two terms are of the $1/\Lambda^3_\chi$ order. Those with (*) is CT $1/\Lambda^3_\chi$ contribution, while that with (**) means octet $1/\Lambda^3_\chi$ contribution (cf diagrams of the Figs.1,3).

$$\begin{aligned}
\mu(p)&=+3.253(1-0.679+0.230^*+0.308^{**})=+3.253(1-0.679+0.538),\\
\mu(n)&=-2.127(1-0.395+0.007^*+0.287^{**})=-2.127(1-0.395+0.294),\\
\mu(\Sigma^+)&=+3.253(1-0.864+0.156^*+0.463^{**})=+3.253(1-0.864+0.619),\\
\mu(\Sigma^-)&=-1.127(1-0.532-0.451^*+1.013^{**})=-1.127(1-0.532-0.562),\quad(11)\\
\mu(\Xi^-)&=-1.127(1-1.167+0.050^*+0.694^{**})=-1.127(1-1.167+0.744),\\
\mu(\Xi^0)&=-2.127(1-1.067+0.094^*+0.561^{**})=-2.127(1-1.067+0.655),
\end{aligned}$$

$$\mu(\Lambda)=-1.063(1-1.040+0.2930^*+0.3241^{**})=-1.063(1-1.040+0.617)=-0.613.\quad(12)$$

The first line for the proton magnetic moment coincides practically with the Eq.(20) of [6]. One can see that chiral perturbation theory expansion is not very impressive as to convergence. Really, counter-terms added to the result of the sum of the diagrams in order to adjust experimental values are, correspondingly,

$\mu(p)^{CT}=0.748$ ( 26.8% *of abs. value of* $\mu(p)=2.793$), $\mu(n)^{CT}=0.015$ ( 0.8% ),
$\mu(\Sigma^+)^{CT}=0.507$ ( 20.6% ), $\mu(\Sigma^-)^{CT}=0.508$ ( 43.8% ),
$\mu(\Xi^-)^{CT}=-0.056$ ( 8.7% ), $\mu(\Xi^0)^{CT}=-0.20$ ( 16.0% ).

That is why it was suggested to include also decuplet contributions [1],[3], [6]. Calculating along the lines of [6] we practically reproduce their results:

$$\mu(p) = +4.673(1 - 0.507 + 0.009^{*} + 0.100^{**}) = +4.673(1 - 0.507 + 0.109) = +2.793$$
$$\mu(n) = -3.188(1 - 0.439 - 0.086^{*} + 0.125^{**}) = -3.188(1 - 0.439 + 0.039) = -1.913,$$
$$\mu(\Sigma^{+}) = +4.673(1 - 0.702 + 0.136^{*} + 0.092^{**}) = +4.673(1 - 0.702 + 0.228) = +2.458,$$
$$\mu(\Sigma^{-}) = -1.484(1 - 0.175 - 0.429^{*} + 0.384^{**}) = -1.484(1 - 0.175 - 0.045) = -1.160, \quad (13)$$
$$\mu(\Xi^{-}) = -1.484(1 - 0.694 - 0.271^{*} + 0.404^{**}) = -1.484(1 - 0.694 + 0.133) = -0.650,$$
$$\mu(\Xi^{0}) = -3.188(1 - 0.925 - 0.169^{*} + 0.486^{**}) = -3.188(1 - 0.925 + 0.317) = -1.250.$$
$$\mu(\Lambda) = -1.594(1 - 1.004 - 0.2371^{*} + 0.6255^{**}) = -1.594(1 - 1.004 + 0.388) = -0.613 \quad (14)$$

Situation with the values of the counter terms seems to become even worse as
$\mu(p)^{CT} = 0.042$ ( 1.5% ), $\mu(n)^{CT} = 0.274$ ( 14.3%),
$\mu(\Sigma^{+})^{CT} = 0.635$ ( 25.9% ), $\mu(\Sigma^{-})^{CT} = 0.637$ ( 54.9% ),
$\mu(\Xi^{-})^{CT} = 0.402$ ( 61.9% ) $\mu(\Xi^{0})^{CT} = 0.539$ ( 43.1% ), $\mu(\Lambda) = 0.378$ (62%).
and we just compare it with the simple-minded unitary model

$$\begin{aligned}
\mu(p) &= 2.579(1 + 0.0829) = +2.793(7.6\%),\\
\mu(n) &= -1.628(1 + 0.175) = -1.913(14.9\%),\\
\mu(\Sigma^{+}) &= 2.579(1 - 0.0470) = +2.458(4.9\%),\\
\mu(\Sigma^{-}) &= -0.951(1 + 0.219) = -1.160(17.9\%),\\
\mu(\Xi^{-}) &= -0.951(1 - 0.316) = -0.650(46.2\%),\\
\mu(\Xi^{0}) &= -1.628(1 - 0.232) = -1.250(30.2\%),\\
\mu(\Lambda) &= -0.814(1 - 0.247) = -0.613(30\%),
\end{aligned} \quad (15)$$

where the value 1 in brackets corresponds to the fit with only $F$ and $D$ constants, while percents in the RHS indicate the value of naive unitary symmetry breaking.

Now we try to improve situation by going to natural units for baryon magnetic moments. Really it is plausible to consider baryon magnetic moments in terms of its proper magnetons. It would be natural to assume that symmetry limit value for every baryon magnetic moment is just its value in the proper magnetons. It does not change one-loop calculations as these are done in terms of chiral theory quantities with all the masses of the baryons in the given unitary multiplet degenerated. But this reasoning changes drastically the values of the terms $\mu_F$, $\mu_D$ and of the resperive counter terms $b$'s. First we calculate corrections in terms of $(1/2m_B)$ with the octet baryon insertions which yields:

$$\mu(p) = 4.0327 - 2.209 + 1.000^{**} - 0.0307^{*} = 2.793;$$

$$\mu(\Sigma^{+}) = 4.0327 - 2.810 + 1.506^{**} + 0.3833^{*} = 3.117;$$

$$\mu(\Sigma^{-}) = -1.3663 + 0.600 - 1.142^{**} + 0.4373^{*} = -1.471;$$

$$\mu(\Xi^{-}) = -1.3663 + 1.3315 - 0.782^{**} - 0.0797^{*} = -0.913;$$

$$\mu(\Xi^{0}) = -2.6663 + 2.270 - 1.193^{**} - 0.1657^{*} = -1.755$$

$$\mu(n) = -2.6663 + 0.840 - 0.610^{**} + 0.5233^{*} = -1.913.$$

$$\mu(\Lambda) = -1.3331 + 1.106 - 0.3445^{**} - 0.1554^{*} = -0.727.$$

The last term with upper * in the sum means contact term. One can see that the CT's are large for $\Sigma^-$ (30%), $n$ (27%). The situation is improved a little, so we try to calculate also decuplet contributions in the loops. The result is:

$$\mu(p) = 4.9822 - 2.369 + 0.467^{**} - 0.2872^{*} = 2.793 \quad (11\%);$$

$$\mu(\Sigma^+) = 4.9822 - 3.280 + 0.430^{**} + 0.9838^{*} = 3.117 \quad (32\%);$$

$$\mu(\Sigma^-) = -1.600 + 0.260 - 0.570^{**} + 0.4388^{*} = -1.471 \quad (25\%);$$

$$\mu(\Xi^-) = -1.600 + 1.030 - 0.600^{**} + 0.2578^{*} = -0.913 \quad (29\%);$$

$$\mu(\Xi^0) = -3.382 + 2.949 - 1.550^{**} + 0.2283^{*} = -1.755 \quad (13\%);$$

$$\mu(n) = -3.382 + 1.400 - 0.399^{**} + 0.4683^{*} = -1.913 \quad (23\%);$$

$$\mu(\Lambda) = -1.691 + 1.600 - 0.997^{**} + 0.361^{*} = -0.727 \quad (50\%).$$

Now we just sum $1/\Lambda_\chi^3$ corrections (loops** + CT*) to arrive at:

$$\mu(p) = 4.9822(1 - 0.4755 + 0.0361) = 2.7932;$$

$$\mu(\Sigma^+) = 4.9822(1 - 0.6583 + 0.2839) = 3.1169;$$

$$\mu(\Sigma^-) = -1.600(1 - 0.1625 + 0.0812) = -1.4699;$$

$$\mu(\Xi^-) = -1.600(1 - 0.6437 + 0.2137) = -0.9121;$$

$$\mu(\Xi^0) = -3.382(1 - 0.8720 + 0.3909) = -1.7549;$$

$$\mu(n) = -3.382(1 - 0.4140 - 0.0201) = -1.9139;$$

$$\mu(\Lambda) = -1.691(1 - 0.9462 + 0.3761) = -0.727.$$

We see that our results look more convergent. Maybe it means that a significient part of the mass corrections now is hidden in the terms $\mu_F, \mu_D$. The overall values of the CT's are about 20-30%, which is less than in previous calculations. The HB$\chi$PT model seems to teach us that it is the mass corrections that are the main sources of discrepancy. But still we see that one-loop chiral corrections do not change the main results of the $SU(3)$ symmetry scheme which gives just similar fit for the octet magnetic moments using very simple assumptions and only two free parameters.

We hope that with some more appropriate treatment of the mass-breaking terms the HB$\chi$PT model may permit a self-consistent description of the baryon magnetic moments.

**Acknowledgments**

We are grateful to A.Ozpineci, V.M.Dubovik, F.Hussain and G.Thompson for discussions.

One of the authors (V.Z.) would like to thank Professor S.Randjbar-Daemi and the High Energy Section of the Abdus Salam ICTP (Trieste, Italy) where the main part of this work was done for the hospitality extended to him.

The financial support of the Abdus Salam International Centre for Theoretical Physics (Trieste, Italy) to V.Z. is gratefully acknowledged.

## References


[1] Jenkins E., Luke M., Manohar A., and Savage M.J.// Phys.Lett.1993.V.B302, P.482; ibid.1996.V.B388, P.866 (E).

[2] Jenkins E., Manohar A.// Phys.Lett.1991.V.B255, P.558 .

[3] Ha P. and Durand L.// Phys.Rev.1998.V.D58, P.093008 .

[4] Durand L. and Ha P.// Phys.Rev.1998.V.D58, P.013010 .

[5] Ha P.// Phys.Rev.1998.V.D58, P.113003 .

[6] Puglia S.J. and Ramsey-Musolf M.J.// Phys.Rev.2000. V.D62, P.093008 .

[7] Banuls M.C., Scimemi I., Bernabeu J., Gimenez V., and Pich A.//Phys.Rev.2000. V.D61, P.074007 .

[8] Coleman S., Glashow S.L.// Phys.Rev.Lett.1961.V.6, P.423 .

[9] Morpurgo G.// Physics (NY) 1965. V.2, P.95.

[10] Georgi H.// Phys.Lett.1990.V.B240, P.447.

[11] Hussein F., Korner J.G., Landgraf J., and Tawfiq S. // Z.Phys.1996.V.C69,P.655.

[12] Bos J.W., Chang D., Lee S.C., Lin Y.C., and Shih H.H.// Chin.J.Phys.1997.V.35, P.150 .

[13] Bukina E.N., Dubovik V.M., Zamiralov V.S.// Vest.Mosk.Un-ta. 2000. N2, P.3 . (in Russian)

[14] Bukina E.N.// J.Mod.Phys.A. 1999.V.14,P.2525.

[15] Gasiorowicz S., Elementary Particle Physics. 1966, John Wiley & Sons. Inc., New York-London-Sydney.

[16] Nguen Van Hieu, Lectures on the Unitary Symmetry Theory of Elementary Particles, Moscow, Atomizdat,1967 (in Russian).

## Appendix A

### Coupling of the Goldstone bosons to octet 1/2 and decuplet 3/2 baryons

Let us write now in detail a unitary structure of the Lagrangian describing an interaction of the pseudo-Goldstone bosons with octet 1/2 baryons. Omitting space-times indices we obtain from the Eq.(3) (see,e.g., [15],[16]):

$$L = DSp\bar{B}\{P,B\} + FSp\bar{B}[P,B].$$

It is convenient to write first the part involving pions and $\eta$-mesons, and then that with kaons:

$$L_{\pi\bar{B}B} = (F+D)[\bar{p}n\pi^+ + \bar{n}p\pi^- + \frac{1}{\sqrt{2}}(\bar{p}p - \bar{n}n)\pi^0] +$$

$$(-F+D)[\overline{\Xi^0}\Xi^-\pi^+ + \overline{\Xi^-}\Xi^0\pi^- - \frac{1}{\sqrt{2}}(\overline{\Xi^0}\Xi^0 - \overline{\Xi^-}\Xi^-)\pi^0]$$

$$+(-\sqrt{2}F)[\overline{\Sigma^+}\Sigma^0\pi^+ - \overline{\Sigma^0}\Sigma^-\pi^+ + \overline{\Sigma^0}\Sigma^+\pi^- - \overline{\Sigma^-}\Sigma^0\pi^- - (\overline{\Sigma^+}\Sigma^+ - \overline{\Sigma^-}\Sigma^-)\pi^0] + \qquad (A1)$$

$$\sqrt{\frac{2}{3}}D[\overline{\vec{\Sigma}}\Lambda\vec{\pi} + \overline{\Lambda}\vec{\Sigma}\vec{\pi} + \overline{\vec{\Sigma}}\vec{\Sigma}\eta - \overline{\Lambda}\Lambda\eta] + \frac{1}{\sqrt{6}}(3F-D)(\bar{p}p+\bar{n}n)\eta - \frac{1}{\sqrt{6}}(3F+D)(\overline{\Xi^0}\Xi^0 + \overline{\Xi^-}\Xi^-)\eta;$$

$$L_{KBB} = (D-F)\left\{\frac{1}{\sqrt{2}}(\overline{\Sigma^0}pK^- - \overline{\Sigma^0}nK^0) + \overline{\Sigma^+}pK^0 + \overline{n}\Sigma^-K^+\right\} +$$

$$(F+D)\left\{\frac{1}{\sqrt{2}}(\overline{\Xi^-}\Sigma^0K^- - \overline{\Xi^0}\Sigma^0K^0) + \overline{\Xi^0}\Sigma^+K^- + \overline{\Xi^-}\Sigma^-K^0\right\} \qquad (A2)$$

$$\frac{1}{\sqrt{6}}(3F+D)\left\{\overline{\Lambda}pK^- + \overline{\Lambda}nK^0\right\} + \frac{1}{\sqrt{6}}(3F-D)\left\{\overline{\Xi^-}\Lambda K^- + \overline{\Xi^0}\Lambda K^0\right\} + H.C.$$

In order to include into the $\pi, K, \eta$-loops also decuplet contributions we write in detail the unitary part of the Lagrangian , describing an interaction of the pseudoscalar mesons with the decuplet 3/2 baryons (omitting space-time indices)(see, e.g.,[15],[16]). With $\mathcal{C} = c^*$ it coincides with the respective part of the Eq.(3):

$$L^* = \mathcal{C}\epsilon_{\tau\alpha\beta}\overline{B^\alpha_\eta}T^{\rho\beta\eta}P^\tau_\rho + H.C. = \frac{\mathcal{C}}{\sqrt{3}}\left\{[2\overline{\Delta^+}p + 2\overline{\Delta^0}n - \overline{\Sigma^{*+}}\Sigma^+ - \overline{\Sigma^{*-}}\Sigma^- - \sqrt{3}\overline{\Sigma^{*0}}\Lambda^0 -\right.$$

$$\overline{\Xi^{*0}}\Xi^0 - \overline{\Xi^{*-}}\Xi^-]\frac{\pi^0}{\sqrt{2}} + [\overline{\Sigma^{*+}}(\frac{1}{\sqrt{2}}\Sigma^0 + \frac{3}{\sqrt{6}}\Lambda^0) - \sqrt{3}\overline{\Delta^{++}}p + \frac{1}{\sqrt{2}}\overline{\Sigma^{*0}}\Sigma^- - \overline{\Delta^+}n + \overline{\Xi^{*0}}\Xi^-]\pi^+ +$$

$$[\sqrt{3}\overline{\Delta^{++}}\Sigma^+ - \sqrt{2}\overline{\Delta^+}\Sigma^0 - \overline{\Delta^0}\Sigma^- + \overline{\Sigma^{*+}}\Xi^0 - \frac{1}{\sqrt{2}}\overline{\Sigma^{*0}}\Xi^-]K^+ +$$

$$[\overline{\Delta^0}p + \overline{\Delta^-}n - \frac{1}{\sqrt{2}}(\overline{\Sigma^{*0}}\Sigma^+ - \overline{\Sigma^{*-}}\Sigma^0) - \frac{3}{\sqrt{6}}\overline{\Sigma^{*-}}\Lambda^0 - \overline{\Xi^{*-}}\Xi^0]\pi^-$$

$$+[\overline{\Delta^+}\Sigma^+ - \sqrt{2}\overline{\Delta^0}\Sigma^0 - \sqrt{3}\overline{\Delta^-}\Sigma^- + \frac{1}{\sqrt{2}}\overline{\Sigma^{*0}}\Xi^0 - \overline{\Sigma^{*-}}\Xi^-]K^0 + [\frac{1}{\sqrt{2}}\overline{\Sigma^{*0}}p - \overline{\Xi^{*0}}\Sigma^+ + \overline{\Sigma^{*-}}n +$$

$$\overline{\Xi^{*-}}(\frac{1}{\sqrt{2}}\Sigma^0 - \frac{3}{\sqrt{6}}\Lambda^0) - \sqrt{3}\overline{\Omega^-}\Xi^0]K^- + [\overline{\Xi^{*0}}(\frac{1}{\sqrt{2}}\Sigma^0 + \frac{3}{\sqrt{6}}\Lambda^0) - \overline{\Sigma^{*-}}p + \overline{\Xi^{*-}}\Sigma^- - \overline{\Sigma^{*0}}n +$$

$$\sqrt{3}\overline{\Omega^-}\Xi^-]\overline{K^0}+[-\overline{\Sigma^{*+}}\Sigma^+ +\overline{\Sigma^{*0}}\Sigma^0+\overline{\Sigma^{*-}}\Sigma^- -\overline{\Xi^{*0}}\Xi^0+\overline{\Xi^{*-}}\Xi^-]\sqrt{\frac{3}{2}}\eta\Bigg\}+H.C. \qquad (A3)$$

**Appendix B**

**On the deduction of the Λ-hyperon magnetic moment**

We can deduce Λ-hyperon magnetic moment from that for the $\Sigma^0$-hyperon using non-linear relations which have their origin in U- and V-spins. Interchanging quarks $d \leftrightarrow s$ one just gets $\Sigma^+$ quark model wave function from that of the proton one, $\Xi^0$ quark model wave function from that of the neutron one and so on. As for $\Sigma^0(ud,s)$, it just changes to $\tilde{\Sigma}^0(us,d)$ which is the state with $U=1, U_3=0$. Similarly, interchanging quarks $u \leftrightarrow s$ one gets $\Xi^-$ quark model wave function from that of the proton and so on. As for $\Sigma^0(ud,s)$, it just changes to $\tilde{\Sigma}^0(ds,u)$ which is the state with $V=1, V_3=0$. It is easy to show that there is a relation between the magnetic moments of the states $\Sigma^0$, $\tilde{\Sigma}^0(us,d)$, $\tilde{\Sigma}^0(ds,u)$ and Λ:

$$2\mu(\tilde{\Sigma}^0(ds,u))+2\mu(\tilde{\Sigma}^0(us,d))-\mu(\Sigma^0)=3\mu(\Lambda). \qquad (B1)$$

So we can perform calculations with the Σ-like quark model wave functions instead of the Λ-like ones. Taking the standard NRQM relation for the $\Sigma^0(ud,s)$ magnetic moments,

$$\mu(\Sigma^0)=\frac{2}{3}\mu_u+\frac{2}{3}\mu_d-\frac{1}{3}\mu_s,$$

we obtain formally

$$\mu(\tilde{\Sigma}^0(ds,u))=\frac{2}{3}\mu_s+\frac{2}{3}\mu_d-\frac{1}{3}\mu_u, \quad \mu(\tilde{\Sigma}^0(us,d))=\frac{2}{3}\mu_s+\frac{2}{3}\mu_u-\frac{1}{3}\mu_d,$$

and Eq.(B1) yields

$$\mu(\Lambda)=\mu_s,$$

as it should be.

In the same way in the $SU(3)_f$ model magnetic moment of any Λ-like hyperon Eq.(7) can be derived from the magnetic moment of the corresponding Σ-like one Eq.(6) as follows:

$$\begin{aligned}\mu(\Sigma^0(qq',h))&=(c_q+c_{q'})F+c_h(F-D),\\ \mu(\tilde{\Sigma}^0(hq',q)&=(c_h+c_{q'})F+c_q(F-D),\\ \mu(\tilde{\Sigma}^0(hq,q')&=(c_h+c_q)F+c_{q'}(F-D),\end{aligned} \qquad (B2)$$

wherefrom upon using Eq.(B1):

$$\mu_{\Lambda(qq'h)}=(c_q+c_{q'})(\mu_F-\frac{2}{3}\mu_D)+c_h(\mu_F+\frac{1}{3}\mu_D). \qquad (B3)$$

Яковлев Станислав Борисович
Замиралов Валерий Семенович

СУММИРОВАНИЕ ДИАГРАММ В НВChPT: НОВЫЕ РЕЗУЛЬТАТЫ ДЛЯ МАГНИТНЫХ МОМЕНТОВ ОКТЕТА БАРИОНОВ

Препринт НИИЯФ МГУ 2003–11/724
Работа поступила в ОНТИ 23.05.2003

**ИД 00545 от 06.12.1999 Издательство**
**Учебно-научного центра довузовского образования**

117246, Москва, ул. Обручева,55А
119992, Москва, Ленинские горы, ГЗ МГУ, Ж-105а
Тел./факс(095)718-6966, 939-3934
e-mail:izdat@abiturcenter.ru
http:/www.abiturcenter.ru

Гигиенический сертификат 77.99.2.925.П.9139.2.00 от 24.02.2000
Налоговые льготы - Общероссийский классификатор продукции
ОК-005-93, том1 - 953000

Заказное. Подписано к печати 23.05.2003 г. Формат 60×90/16
Бумага офсетная 2. Усл.печ.л. 2,0
Тираж 25 экз. Заказ 377
Отпечатано в Мини-типографии УНЦ ДО
в полном соответствии с качеством
предоставленного оригинал-макета