E.E. Boos, Yu.S. Mikhailov,
M.N. Smolyakov, I.P. Volobuev

PHYSICAL DEGREES OF FREEDOM IN STABILIZED BRANE WORLD MODELS

Preprint SINP MSU 2005-30/796

D.V. SKOBELTSYN INSTITUTE OF NUCLEAR PHYSICS

E.E. Boos, Yu.S. Mikhailov[1],
M.N. Smolyakov, I.P. Volobuev

PHYSICAL DEGREES OF FREEDOM IN STABILIZED BRANE WORLD MODELS

Preprint SINP MSU 2005-30/796

[1]Physics Department, M.V. Lomonosov Moscow State University

УДК 531.5(075.8)

Boos E.E., Mikhailov Yu.S., Smolyakov M.N., Volobuev I.P.

e-mail: volobuev@theory.sinp.msu.ru

Physical degrees of freedom in stabilized brane world models

Preprint SINP MSU 2005-30/796

Abstract

We consider brane world models with interbrane separation stabilized by the Goldberger-Wise scalar field. For arbitrary background, or vacuum configurations of the gravitational and scalar fields in such models, we construct the second variation Lagrangian, study its gauge invariance, find the corresponding equations of motion and decouple them in a suitable gauge. We also derive an effective four-dimensional Lagrangian for such models, which describes the massless graviton, a tower of massive gravitons and a tower of massive scalars. It is shown that for a special choice of the background solution the masses of the graviton excitations may be of the order of 1 TeV, the radion mass of the order of 100 GeV, the inverse size of the extra dimension being 1 TeV. In this case the inverse coupling of the radion to matter on the negative tension brane is of the order of 50 TeV.

Боос Э.Э., Волобуев И.П., Михайлов Ю.С., Смоляков М.Н.

Физические степени свободы в стабилизированных моделях мира на бране

Препринт НИИЯФ МГУ 2005-30/796

Аннотация

Рассмотрены модели мира на бране, в которых расстояние между бранами стабилизировано с помощью скалярного поля Гольдбергера-Вайза. Для произвольной вакуумной конфигурации гравитационного и скалярного поля в таких моделях построен лагранжиан второй вариации, изучена его калибровочная инвариантность, найдены и расцеплены в подходящей калибровке соответствующие уравнения движения. Также получен эффективный четырехмерный лагранжиан для таких моделей, описывающий безмассовый гравитон, набор массивных гравитонов и набор массивных скалярных полей. Показано, что при определенном выборе фонового решения массы гравитационных возбуждений могут быть порядка 1 ТэВ, масса радиона – порядка 100 ГэВ, а обратный размер дополнительного измерения оказывается порядка 1 ТэВ. В этом случае обратная константа связи редиона с материей на бране с отрицательным натяжением оказывается порядка 50 ТэВ.

1 Introduction

Brane world models and their phenomenology have been widely discussed in the last years [1, 2]. One of the most interesting brane world models is the Randall-Sundrum model with two branes, - the RS1 model [3]. This model solves the hierarchy problem due to the warp factor in the metric and predicts an interesting new physics in the TeV range of energies. A flaw of the RS1 model is the presence of a massless scalar mode, called the radion, which arises due to the fluctuations of the branes with respect to each other. Its interactions contradict the existing experimental data, and in order the model be phenomenologically acceptable the radion must acquire a mass, which is equivalent to the stabilization of the brane separation distance. The latter can be achieved by introducing a five-dimensional scalar field with a bulk and brane potentials, whose vacuum energy has a minimum for a certain interbrane distance [4]. A disadvantage of the approach proposed in [4] is that the backreaction of the scalar field on the background metric is not taken into account. This problem is solved in the model proposed in [5].

Nevertheless, almost all the papers on the phenomenology of the RS1 model consider the unstabilized model, just putting the radion mass by hand. In the present paper we are going to show that such an approach is inconsistent, because the backreaction of the scalar field on the metric leads to a renormalization of the parameters of the RS1 model. In particular, the coupling of the radion to matter on the negative tension brane turns out to be smaller, than in the unstabilized model.

2 Linearized gravity in stabilized brane world models

Let us denote the coordinates in five-dimensional space-time $E = M_4 \times S^1/Z_2$ by $\{x^M\} \equiv \{x^\mu, y\}$, $M = 0, 1, 2, 3, 4$, $\mu = 0, 1, 2, 3$, the coordinate $x^4 \equiv y, \quad -L \leq y \leq L$ parameterizing the fifth dimension. It forms the orbifold, which is realized as the circle of the circumference $2L$ with the points y and $-y$ identified. Correspondingly, the metric g_{MN} and the scalar field ϕ satisfy the orbifold symmetry conditions

$$\begin{aligned} g_{\mu\nu}(x,-y) = g_{\mu\nu}(x,y), \quad g_{\mu 4}(x,-y) = -g_{\mu 4}(x,y), \\ g_{44}(x,-y) = g_{44}(x,y), \quad \phi(x,-y) = \phi(x,y). \end{aligned} \tag{1}$$

The branes are located at the fixed points of the orbifold, $y = 0$ and $y = L$.

The action of stabilized brane world models can be written as

$$S = S_g + S_\phi, \tag{2}$$

where S_g and S_ϕ are given by

$$\begin{aligned} S_g &= 2M^3 \int d^4x \int_{-L}^{L} dy R\sqrt{-g}, \\ S_\phi &= -\int d^4x \int_{-L}^{L} dy \left(\frac{1}{2} g^{MN} \partial_M \phi \partial_N \phi + V(\phi)\right) \sqrt{-g} - \\ &- \int_{y=0} \sqrt{-\tilde{g}} \lambda_1(\phi) d^4x - \int_{y=L} \sqrt{-\tilde{g}} \lambda_2(\phi) d^4x. \end{aligned} \tag{3}$$

Here $V(\phi)$ is a bulk scalar field potential and $\lambda_{1,2}(\phi)$ are brane scalar field potentials, $\tilde{g} = det\tilde{g}_{\mu\nu}$, and $\tilde{g}_{\mu\nu}$ denotes the metric induced on the branes. The signature of the metric g_{MN} is chosen to be $(-,+,+,+,+)$.

The standard ansatz for the metric and the scalar field, which preserves the Poincaré invariance in any four-dimensional subspace $y = const$, looks like

$$ds^2 = e^{-2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2 \equiv \gamma_{MN}(y)dx^M dx^N, \quad (4)$$

$$\phi(x,y) = \phi(y), \quad (5)$$

$\eta_{\mu\nu}$ denoting the flat Minkowski metric. If one substitutes this ansatz into the equations corresponding to action (2), one gets a rather complicated system of nonlinear differential equations for functions $A(y), \phi(y)$:

$$\begin{gathered} \tfrac{\delta V}{\delta\phi} + \tfrac{\delta\lambda_1}{\delta\phi}\delta(y) + \tfrac{\delta\lambda_2}{\delta\phi}\delta(y-L) = -4A'\phi' + \phi'' \quad (6) \\ 12M^3(A')^2 + \tfrac{1}{2}(V - \tfrac{1}{2}(\phi')^2) = 0 \\ \tfrac{1}{2}\left(\tfrac{1}{2}(\phi')^2 + V + \lambda_1\delta(y) + \lambda_2\delta(y-L)\right) = -2M^3\left(-3A'' + 6(A')^2\right) \end{gathered}$$

Here $' = \partial_4 \equiv \partial/\partial y$.

Suppose we have a solution $A(y), \phi(y)$ to this system for an appropriate choice of the parameters of the potentials such that the interbrane distance is stabilized and is equal to L. It means that the vacuum energy of the scalar field has a minimum for this value of the interbrane distance.

Now the linearized theory is obtained by representing the metric and the scalar field as

$$g_{MN}(x,y) = \gamma_{MN}(y) + \frac{1}{\sqrt{2M^3}}h_{MN}(x,y), \quad (7)$$

$$\phi(x,y) = \phi(y) + \frac{1}{\sqrt{2M^3}}f(x,y), \quad (8)$$

substituting this representation into action (2) and keeping the terms of the second order in h_{MN} and f. The Lagrangian of this action is called the second variation Lagrangian and has the form

$$\begin{aligned} \frac{\mathcal{L}}{\sqrt{-\gamma}} = &-\tfrac{1}{4}(\bigtriangledown_S h_{MN}\bigtriangledown^S h^{MN} + 2\bigtriangledown_N h\bigtriangledown_M h^{MN} - 2\bigtriangledown_M h^{MN}\bigtriangledown^S h_{SN} - \quad (9) \\ &- \bigtriangledown_S h\bigtriangledown^S h) + (A')^2\left(\tfrac{7}{2}h_{MN}h^{MN} - hh\right) - \\ &-A''\left(h_{MN}h^{MN} - \tfrac{1}{2}\tilde{h}h + \tfrac{1}{2}h_{M\nu}h^{M\nu}\right) + \tfrac{1}{4M^3}\Big[\tfrac{V}{2}\left(h_{MN}h^{MN} - \tfrac{1}{2}hh\right) + \\ &+\tfrac{1}{2}\left(h_{\mu\nu}h^{\mu\nu} - \tfrac{1}{2}\tilde{h}\tilde{h}\right)[\lambda_1\delta(y) + \lambda_2\delta(y-L)] + \\ &+\tfrac{1}{2}(\phi')^2\left(-\tfrac{1}{4}hh + \tfrac{1}{2}h_{MN}h^{MN} + hh_{44} - 2h_{4M}h^{4M}\right) - \\ &-f\left(h\tfrac{\delta V}{\delta\phi} + \tilde{h}[\tfrac{\delta\lambda_1}{\delta\phi}\delta(y) + \tfrac{\delta\lambda_2}{\delta\phi}\delta(y-L)]\right) - f'\phi' h + 2\partial_M f\phi' h^{M4} - \\ &-\partial^M f\partial_M f - f^2\left(\tfrac{\delta^2 V}{\delta\phi^2} + \tfrac{\delta^2\lambda_1}{\delta\phi^2}\delta(y) + \tfrac{\delta^2\lambda_2}{\delta\phi^2}\delta(y-L)\right)\Big]. \end{aligned}$$

Here $h = \gamma_{MN}h^{MN}$, $\tilde{h} = \gamma_{\mu\nu}h^{\mu\nu}$, ϕ stands for the background solution and $\bigtriangledown_M$ denotes the covariant derivative with respect to metric γ_{MN}. The normalization of fields in (7), (8) and

(9) differs from the one adopted in [6]. It is more convenient in the Lagrangian approach and does not effect the equations of motion.

Varying the action built with this Lagrangian and taking into account background field equations (6), we arrive at the following equations of motion for the fluctuations of metric:

1. $\mu\nu$-component

$$\begin{gathered}\tfrac{1}{2}(\partial_\sigma\partial^\sigma h_{\mu\nu}-\partial_\mu\partial^\sigma h_{\sigma\nu}-\partial_\nu\partial^\sigma h_{\sigma\mu}+\partial_4\partial_4 h_{\mu\nu})+\tfrac{1}{2}\partial_\mu\partial_\nu\tilde{h}+\tfrac{1}{2}\partial_\mu\partial_\nu h_{44}\quad-\\ -\tfrac{1}{2}\partial_4(\partial_\mu h_{4\nu}+\partial_\nu h_{4\mu})+A'(\partial_\mu h_{4\nu}+\partial_\nu h_{4\mu})+\tfrac{1}{2}\gamma_{\mu\nu}\Big(-\partial_4\partial_4\tilde{h}-\\ -\partial_\sigma\partial^\sigma h_{44}-\partial_\sigma\partial^\sigma\tilde{h}+4A'\partial_4\tilde{h}-3A'\partial_4 h_{44}+\partial_\sigma\partial_\tau h^{\sigma\tau}+2\partial^\sigma\partial_4 h_{\sigma 4}-\\ -4A'\partial^\sigma h_{4\sigma}\Big)-h_{\mu\nu}(2A'^2-A'')+\tfrac{3}{2}h_{44}\gamma_{\mu\nu}(4A'^2-A'')-\\ -\tfrac{1}{4M^3}\gamma_{\mu\nu}\left(f'\phi'+f(-4A'\phi'+\phi'')\right)=0;\end{gathered}\tag{10}$$

2. $\mu 4$-component

$$\partial_4(\partial_\mu\tilde{h}-\partial^\nu h_{\mu\nu})+\partial^\nu(\partial_\nu h_{\mu 4}-\partial_\mu h_{\nu 4})+3A'\partial_\mu h_{44}+\frac{1}{2M^3}\partial_\mu f\phi'=0;\tag{11}$$

3. 44-component

$$\partial^\mu(\partial^\nu h_{\mu\nu}-\partial_\mu\tilde{h})-6A'\partial^\mu h_{\mu 4}+3A'\partial_4\tilde{h}+\frac{1}{2M^3}\left(-h_{44}V-f\frac{\delta V}{\delta\phi}+f'\phi'\right)=0;\tag{12}$$

4. equation for the field f

$$\begin{gathered}h_{44}\left(\tfrac{\delta\lambda_1}{\delta\phi}\delta(y)+\tfrac{\delta\lambda_2}{\delta\phi}\delta(y-L)\right)-2h_{44}(-4A'\phi'+\phi'')+\\ +\phi'\partial_4\tilde{h}-\phi'\partial_4 h_{44}-2\phi'\partial^\mu h_{\mu 4}-8A'\partial_4 f+2\partial_M\partial^M f-\\ -2f\left(\tfrac{\delta^2 V}{\delta\phi^2}+\tfrac{\delta^2\lambda_1}{\delta\phi^2}\delta(y)+\tfrac{\delta^2\lambda_2}{\delta\phi^2}\delta(y-L)\right)=0.\end{gathered}\tag{13}$$

We will also use the following auxiliary equation, which is obtained by contracting the indices in the $\mu\nu$-equation:

$$\begin{gathered}\partial^\mu\partial^\nu h_{\mu\nu}-\partial^\mu\partial_\mu(\tilde{h}+\tfrac{3}{2}h_{44})-6A'\partial_4(h_{44}-\tilde{h})-\tfrac{3}{2}\partial_4\partial_4\tilde{h}-6A'\partial^\mu h_{\mu 4}+\\ +3\partial^\mu\partial_4 h_{\mu 4}+6h_{44}(4A'^2-A'')-\tfrac{1}{M^3}(f'\phi'+f(-4A'\phi'+\phi''))=0.\end{gathered}\tag{14}$$

These equations are invariant under the gauge transformations

$$\begin{gathered}h^{(\prime)}_{\mu\nu}(x,y)=h_{\mu\nu}(x,y)-(\partial_\mu\xi_\nu+\partial_\nu\xi_\mu-2\gamma_{\mu\nu}\partial_4 A\xi_4),\\ h^{(\prime)}_{\mu 4}(x,y)=h_{\mu 4}(x,y)-(\partial_\mu\xi_4+\partial_4\xi_\mu+2\partial_4 A\xi_\mu),\\ h^{(\prime)}_{44}(x,y)=h_{44}(x,y)-2\partial_4\xi_4,\end{gathered}\tag{15}$$

$$f^{(\prime)}(x,y)=f(x,y)-\partial_4\phi\xi_4,\tag{16}$$

provided $\xi_M(x,y)$ satisfy the orbifold symmetry conditions

$$\xi_\mu(x,-y)=\xi_\mu(x,y),\quad \xi_4(x,-y)=-\xi_4(x,y).$$

These gauge transformations are a generalization of the gauge transformations in the unstabilized RS1 model [2, 8]. We will use them to isolate the physical degrees of freedom of the fields h_{MN} and f. Let us show that the gauge transformations with function ξ_4 allow one to impose the gauge condition

$$(e^{-2A}h_{44})'-\frac{1}{3M^3}e^{-2A}\phi' f=0. \tag{17}$$

Really, eqs. (15), (16) imply the following equation for ξ_4

$$\partial_4\partial_4\xi_4-2A'\partial_4\xi_4-\frac{1}{6M^3}(\phi')^2\xi_4=-\frac{1}{2}(\partial_4 h_{44}-2A'h_{44}-\frac{1}{3M^3}\phi' f).$$

The standard theory of differential equations demands the coefficient in front of $\partial_4\xi_4$ to be continuous, though in our case it is not so. To cure this flaw, we rewrite the equation as follows:

$$\begin{aligned}&\partial_4\partial_4(\xi_4 e^{-A})+\xi_4 e^{-A}\left(A''-(A')^2-\tfrac{1}{6M^3}(\phi')^2\right)= \\ &=-\tfrac{1}{2}e^{-A}(\partial_4 h_{44}-2A'h_{44}-\tfrac{1}{3M^3}\phi' f)\equiv w(x,y).\end{aligned} \tag{18}$$

The functions $(A')^2$ and $(\phi')^2$ are smooth functions of y. Although function A'' has δ-like singularities at $y=0, y=L$, the singular terms drop from the equation, because ξ_4 is equal to zero at these points. Thus, the factor in front of $\xi_4 e^{-A}$ is a smooth function, and the equation for $\xi_4 e^{-A}$ in the interval $[-L,L]$ can be treated by the standard methods. In accordance with the general theory, the homogeneous equation, corresponding to (18), has two independent solutions in the interval $[0,L]$; we denote them $\chi_1(y)$ and $\chi_2(y)$. Now we can use the Green function method to find ξ_4, which gives:

$$\begin{aligned}\xi_4(x,y)=e^A\int_0^L\frac{-\chi_1(y)\chi_2(z)+\chi_2(y)\ chi_1(z)}{\chi_1'(z)\chi_2(z)-\chi_2'(z)\chi_1(z)}\theta(y-z)w(x,z)dz- \\ -e^A\frac{\chi_1(y)\chi_2(0)-\chi_2(y)\chi_1(0)}{\chi_1(L)\chi_2(0)-\chi_2(L)\chi_1(0)}\int_0^L\frac{-\chi_1(L)\chi_2(z)+\chi_2(L)\chi_1(z)}{\chi_1'(z)\chi_2(z)-\chi_2'(z)\chi_1(z)}w(x,z)dz.\end{aligned} \tag{19}$$

It is not difficult to check that the obtained function ξ_4 is equal to zero at the ends of the interval and therefore can be continued to an odd function on $[-L,L]$. Thus, gauge condition (17) really exists. We also note that this relation was obtained in [7] from the equation for $\mu 4$-component, in which only the scalar degrees of freedom were retained. Similar to the case of the unstabilized RS1 model, the gauge transformations with functions ξ_μ allow one to impose the gauge $h_{\mu 4}(x,y)=0$, after which there remain the gauge transformations satisfying

$$\partial_4(e^{2A}\xi_\mu)=0. \tag{20}$$

Thus, we can use the gauge

$$\begin{aligned}(e^{-2A}h_{44})'-\frac{1}{3M^3}e^{-2A}\phi' f=0,\\ h_{\mu 4}=0.\end{aligned} \tag{21}$$

Next we represent the gravitational field as

$$h_{\mu\nu} = b_{\mu\nu} + \frac{1}{4}\gamma_{\mu\nu}\tilde{h}, \tag{22}$$

with $b_{\mu\nu}$ being a traceless tensor field ($\gamma^{\mu\nu}b_{\mu\nu} = 0$).

Substituting gauge conditions (21) and representation (22) into the $\mu 4$-equation and into contracted $\mu\nu$-equation (14), we get:

$$-\partial_4(\partial^\nu b_{\mu\nu}) + \tfrac{3}{4}\partial_\mu\partial_4(\tilde{h} + 2h_{44}) = 0, \tag{23}$$

$$\partial^\mu\partial^\nu b_{\mu\nu} - \tfrac{3}{4}\partial_\rho\partial^\rho\tilde{h} - \tfrac{3}{2}\partial_\rho\partial^\rho h_{44} - \tfrac{3}{2}\tfrac{\partial^2}{\partial y^2}\tilde{h} + \tag{24}$$

$$+6A'\partial_4\tilde{h} - 3\tfrac{\partial^2}{\partial y^2}h_{44} + 12A'\partial_4 h_{44} = 0.$$

Equation (23) suggest the substitution $\tilde{h} = -2h_{44}$, which allows one to decouple the equations for the fields $b_{\mu\nu}, h_{44}$ and f. Really, as a result of this substitution equations (23), (24) take the form

$$\partial_4(\partial^\nu b_{\mu\nu}) = 0, \tag{25}$$

$$\partial^\mu\partial^\nu b_{\mu\nu} = 0. \tag{26}$$

It is not difficult to check that the residual gauge transformations (20) are sufficient to impose the gauge [8]

$$\partial^\nu b_{\mu\nu} = 0,$$

in which the former equations are satisfied identically.

Thus, in what follows we will be working in the gauge

$$(e^{-2A}h_{44})' - \frac{1}{3M^3}e^{-2A}\phi' f = 0, \tag{27}$$

$$h_{\mu 4} = 0,$$

$$\tilde{b} = \gamma^{\mu\nu}b_{\mu\nu} = 0, \quad \partial^\nu b_{\mu\nu} = 0,$$

the residual gauge transformations now being

$$\xi_\mu = e^{-2A}\epsilon_\mu(x), \qquad \partial^\nu\epsilon_\nu(x) = 0, \qquad \Box\epsilon_\nu = 0. \tag{28}$$

Obviously, after the substitution $\tilde{h} = -2h_{44}$ contracted $\mu\nu$-equation (14) and the $\mu 4$-equation are satisfied identically in this gauge. Equation (10) for the $\mu\nu$-component reduces to an equation for a transverse-traceless tensor field $b_{\mu\nu}(x, y)$:

$$\frac{1}{2}\left(e^{2A(y)}\Box b_{\mu\nu} + \frac{\partial^2 b_{\mu\nu}}{\partial y^2}\right) - b_{\mu\nu}\left(2(A')^2 - A''\right) = 0. \tag{29}$$

This equation does not include the background scalar filed $\phi(y)$ and is absolutely analogous to the corresponding equation in the unstabilized RS1 model.

In order to find equations for the scalar field h_{44}, we have to solve gauge condition (27) with respect to f and to substitute the latter into equations (12), (13). This can be done either using a regularization $(sign(y))^2 = 1$, or restricting the equations to the interval $(0, L)$

and taking into account their singular terms with the help of boundary conditions. The latter technique turns out to be simpler and we will use it.

Equation for 44-component (12) simplifies considerably, when rewritten in the interval $(0, L)$ in terms of a new function $g = e^{-2A(y)} h_{44}(x, y)$ and using the expression for the potential V in terms of A and ϕ (6):

$$g'' + 2g' \left(A' - \frac{\phi''}{\phi'} \right) - \frac{(\phi')^2}{6M^3} g + \partial_\mu \partial^\mu g = 0. \tag{30}$$

Let us note that substitution (22) and gauge condition (27), which decouple the equations of motion, in terms of g take the form:

$$h_{\mu\nu} = b_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} g, \quad h_{44} = e^{2A(y)} g, \tag{31}$$

$$g' - \frac{1}{3M^3} e^{-2A} \phi' f = 0, \tag{32}$$

$$h_{\mu 4} = 0, \quad \tilde{b} = \gamma_{\mu\nu} b^{\mu\nu} = 0, \quad \partial^\nu b_{\mu\nu} = 0. \tag{33}$$

Gauge condition (32) solved for f in the interval $(0, L)$, looks like

$$f = 3M^3 \frac{e^{2A}}{\phi'} g'.$$

Substituting this expression for f into eq. (13) gives an equation, which is obtained by differentiating eq. (30) with respect to y, and boundary conditions on the branes:

$$\begin{aligned} \left(\tfrac{1}{2} \tfrac{\delta^2 \lambda_1}{\delta \phi^2} - \tfrac{\phi''}{\phi'} \right) g' + \partial_\mu \partial^\mu g|_{y=+0} = 0, \\ \left(\tfrac{1}{2} \tfrac{\delta^2 \lambda_2}{\delta \phi^2} + \tfrac{\phi''}{\phi'} \right) g' - \partial_\mu \partial^\mu g|_{y=L-0} = 0. \end{aligned} \tag{34}$$

Thus, our gauge choice and the substitution enabled us to decouple the equations of motion. This means that the fluctuations of the metric and of the scalar field against any background given by a solution to eqs. (6) are described by two fields, – tensor field $b_{\mu\nu}(x, y)$ and scalar field $g(x, y)$. Their classical equations of motion are given by equations (29) and (30), (34) respectively.

3 Mode decompositions

Let us study first the modes of the tensor field $b_{\mu\nu}(x, y)$, which satisfies equation (29). Substituting into this equation

$$b_{\mu\nu}(x, y) = c_{\mu\nu} e^{ipx} \psi_n(y), \quad c_{\mu\nu} = const, \quad p^2 = -m_n^2,$$

restricting it to the interval $(0, L)$ and replacing the singular terms by the boundary conditions, we get:

$$\begin{aligned} \tfrac{d^2 \psi_n}{dy^2} - 2((A')^2 - A'')\psi_n = -m_n^2 e^{2A} \psi_n, \\ \psi_n' + 2A' \psi_n|_{y=+0} = \psi_n' + 2A' \psi_n|_{y=L-0} = 0. \end{aligned} \tag{35}$$

The boundary conditions suggest a substitution $\psi_n = \exp(-2A)\omega_n$, which turns this equation into

$$\begin{aligned} \tfrac{d}{dy}\left(e^{-4A}\omega_n'\right) &= -m_n^2 e^{-2A}\omega_n, \\ \omega_n'|_{y=+0} &= \omega_n'|_{y=L-0} = 0. \end{aligned} \tag{36}$$

We see that the eigenfunctions ω_n are solutions of a Sturm-Liouville problem with von Neumann boundary conditions. In accordance with the general theory [9], the problem at hand has no negative eigenvalues for arbitrary A, only one zero eigenvalue, corresponding to $\omega_0 = const$, and an infinite number of positive eigenvalues, asymptotically given by the formula

$$m_n^2 = \frac{\pi^2 n^2}{l^2}, \quad l = \int_0^L e^{A(y)} dy. \tag{37}$$

This formula should be specified for finding the masses on different branes. We recall that the masses of excitations on each brane should be calculated in the Galilean coordinates [6, 8], for which $A(y)$ is equal to zero on the corresponding brane (we recall that coordinates are called Galilean if $g_{\mu\nu} = (-1, 1, 1, 1)$ [10]). Thus, the latter formula can be explicitly adopted for calculating masses on the branes as follows:

$$l = \textstyle\int_0^L e^{(A(y)-A(0))} dy \quad \text{for the brane at} \quad y = 0, \tag{38}$$

$$l = \textstyle\int_0^L e^{(A(y)-A(L))} dy \quad \text{for the brane at} \quad y = L, \tag{39}$$

which is valid for an arbitrary $A(y)$ satisfying equations (6), because they define it up to an additive constant.

The eigenfunctions $\{\psi_n(y)\}$ of eigenvalue problem (35) build a complete orthonormal set, the eigenfunction of the zero mode being

$$\psi_0(y) = N e^{-2A(y)}. \tag{40}$$

Expanding $b_{\mu\nu}$ in this system

$$b_{\mu\nu} = \sum_{n=0}^{\infty} b_{\mu\nu}^n(x)\psi_n(y), \tag{41}$$

we get four-dimensional tensor fields $b_{\mu\nu}^n(x)$ with definite masses. An important point is that due to the form of the zero mode eigenfunction residual gauge transformations (28) act only on the massless field $b_{\mu\nu}^0(x)$ and provide the correct number of degrees of freedom of the massless graviton [8].

In order to find the mass spectrum of the scalar particles described by equation (30) let us substitute

$$g(x, y) = e^{ipx} g_n(y), \quad p^2 = -\mu_n^2,$$

into this equation. As a result, the equation and the boundary conditions for $g_n(y)$ take the form:

$$g_n'' + 2A' g_n' - 2\tfrac{\phi''}{\phi'} g_n' - \tfrac{(\phi')^2}{6M^3} g_n = -\mu_n^2 e^{2A} g_n, \tag{42}$$

$$\left(\tfrac{1}{2}\tfrac{\delta^2\lambda_1}{\delta\phi^2} - \tfrac{\phi''}{\phi'}\right) g_n' + \mu_n^2 e^{2A} g_n|_{y=+0} = 0, \tag{43}$$

$$\left(\tfrac{1}{2}\tfrac{\delta^2\lambda_2}{\delta\phi^2} + \tfrac{\phi''}{\phi'}\right) g_n' - \mu_n^2 e^{2A} g_n|_{y=L-0} = 0. \tag{44}$$

Let us write equation (42) in the Sturm-Liouville form:

$$\frac{d}{dy}\left(\frac{e^{2A}}{(\phi')^2}g_n'\right) - \frac{e^{2A}}{6M^3}g_n = -\mu_n^2 g_n \frac{e^{4A}}{(\phi')^2}. \tag{45}$$

It is not difficult to see that the operator in the eigenvalue problem for this equation with boundary conditions (43), (44) is not self-adjoint. Nevertheless, these boundary conditions look like the usual Sturm-Liouville boundary conditions and lead to a number of general assertions about the spectrum and the eigenfunctions of the problem at hand.

Multiplying equation (45) by $\bar{g}_n$, then integrating over $(0, L)$ and integrating by parts in the term with derivatives, we get:

$$\mu_n^2\left(\int_0^L \frac{e^{4A}}{(\phi')^2}|g_n|^2dy + \left(\frac{1}{2}\frac{\delta^2\lambda_1}{\delta\phi^2} - \frac{\phi''}{\phi'}\right)^{-1}\frac{e^{4A}}{(\phi')^2}|g_n|^2|_{y=+0} + \left(\frac{1}{2}\frac{\delta^2\lambda_2}{\delta\phi^2} + \frac{\phi''}{\phi'}\right)^{-1}\frac{e^{4A}}{(\phi')^2}|g_n|^2|_{y=L-0}\right) =$$
$$= \frac{1}{6M^3}\int_0^L e^{2A}|g_n|^2dy + \int_0^L \frac{e^{2A}}{(\phi')^2}|g_n'|^2dy.$$

This means that if

$$\left(\frac{1}{2}\frac{\delta^2\lambda_1}{\delta\phi^2} - \frac{\phi''}{\phi'}\right)|_{y=+0} > 0, \quad \left(\frac{1}{2}\frac{\delta^2\lambda_2}{\delta\phi^2} + \frac{\phi''}{\phi'}\right)|_{y=L-0} > 0, \tag{46}$$

all the eigenvalues of the eigenproblem are real and positive. The standard technique for proving the orthogonality of eigenfunctions gives in this case for $m \neq n$

$$-\int_0^L \frac{e^{4A}}{(\phi')^2}\bar{g}_m g_n dy = \tag{47}$$
$$= \left(\frac{1}{2}\frac{\delta^2\lambda_1}{\delta\phi^2} - \frac{\phi''}{\phi'}\right)^{-1}\frac{e^{4A}}{(\phi')^2}\bar{g}_m g_n|_{y=+0} + \left(\frac{1}{2}\frac{\delta^2\lambda_2}{\delta\phi^2} + \frac{\phi''}{\phi'}\right)^{-1}\frac{e^{4A}}{(\phi')^2}\bar{g}_m g_n|_{y=L-0}.$$

Thus, the eigenfunctions of this problem, corresponding to different eigenvalues, are not orthogonal with respect to the weight suggested by the form of equation (45).

It is very difficult to prove rigorously that this set of eigenfunctions is complete. We can just argue that due to the special form of boundary conditions (43), (44) the set of eigenfunctions of the eigenvalue problem under consideration is complete. Really, for the parameters of the scalar field potential $\frac{1}{2}\frac{\delta^2\lambda_{1,2}}{\delta\phi^2} \to \infty$ the eigenvalues drop from the boundary conditions, and the operator in the equation becomes self-adjoint. Therefore, for $\frac{1}{2}\frac{\delta^2\lambda_{1,2}}{\delta\phi^2} \to \infty$ the eigenfunctions of the problem under consideration go to the eigenfunctions of a Sturm-Liouville problem with a self-adjoint operator, which build a complete orthogonal set. The orthogonality of the eigenfunctions for $\frac{1}{2}\frac{\delta^2\lambda_{1,2}}{\delta\phi^2} \to \infty$ can be also seen in formula (47). Thus, we assume that the eigenfunctions $\{g_n(y)\}$ of the problem (42) - (44) form a complete denumerable non-orthogonal set. We would also like to note that for $\mu_n^2 \to \infty$ boundary conditions (43), (44) go to $g_n(0) = g_n(L) = 0$, and therefore for large m or n the integral in (47) is close to zero. Such a set of functions may be called asymptotically orthogonal.

It is easy to understand that the eigenfunctions $g_n(y)$ can be chosen to be real. Then five-dimensional scalar field $g(x, y)$ can be expanded in this system as

$$g(x, y) = \sum_{n=1}^{\infty} \varphi_n(x) g_n(y), \tag{48}$$

where the four dimensional real scalar fields $\varphi_n(x)$ have masses μ_n^2.

Now we can find the effective four-dimensional action for the system. Substituting (22), (31)-(33) into Lagrangian (9), we find that the tensor field and the scalar field Lagrangians decouple. Then substituting expansions (41), (48) into these Lagrangians and using equations (35), (42), assuming the eigenfunctions $\{\psi_n\}$ and $\{g_n\}$ to be appropriately normalized and integrating over the coordinate of the extra dimension, we find that the reduced tensor field Lagrangian is a sum of the standard four-dimensional Lagrangians of tensor fields with masses m_n, whereas for the scalar field Lagrangian we get

$$\mathcal{L}_{scalar} = -\frac{3}{4}\sum_{nk}\left[\eta^{\mu\nu}\partial_\mu\varphi_n\partial_\nu\varphi_k + \mu_k^2\varphi_n\varphi_k\right]\int_0^L dy e^{2A}\left(g_n g_k + \frac{6M^3}{(\phi')^2}g_n' g_k'\right). \qquad (49)$$

Now consider the integral over dy. Integrating by parts the term with derivatives, using equation (45) and boundary conditions (43), (44), we arrive at the result

$$6\mu_n^2 M^3\left(\int_0^L \frac{e^{4A}}{(\phi')^2}g_n g_k dy + \left(\frac{1}{2}\frac{\delta^2\lambda_1}{\delta\phi^2} - \frac{\phi''}{\phi'}\right)^{-1}\frac{e^{4A}}{(\phi')^2}g_n g_k|_{y=+0} + \right.$$
$$\left. + \left(\frac{1}{2}\frac{\delta^2\lambda_2}{\delta\phi^2} + \frac{\phi''}{\phi'}\right)^{-1}\frac{e^{4A}}{(\phi')^2}g_n g_k|_{y=L-0}\right).$$

For real $g_n(y)$ equation (47) now implies that this expression is equal to zero for $n \neq k$. Assuming the eigenfunctions $\{g_n\}$ to be appropriately normalized, we find that the reduced scalar field Lagrangian is also the standard Lagrangian, and the complete reduced action is

$$S_{eff} = -\frac{1}{4}\sum_k\int dx\left(\partial^\sigma b^{k,\mu\nu}\partial_\sigma b^k_{\mu\nu} + m_k^2 b^{k,\mu\nu}b^k_{\mu\nu}\right) - \frac{1}{2}\sum_k\int dx\left(\partial_\nu\varphi_k\partial^\nu\varphi_k + \mu_k^2\varphi_k\varphi_k\right). \quad (50)$$

We see that the effective Lagrangian of the tensor fields coincides with the one of the unstabilized model [8].

Now we are able to find the couplings of the four-dimensional fields $b^n_{\mu\nu}(x)$ and $\varphi_n(x)$, to matter on the branes, which are defined by the coupling of the fluctuations of the five-dimensional gravitational field $h_{\mu\nu}$ to matter on the branes. The latter is given by

$$\frac{1}{\sqrt{8M^3}}\int_{B_1} h_{\mu\nu}(x,0)T^{(1)}_{\mu\nu}\sqrt{-det\gamma_{\mu\nu}(0)}dx + \frac{1}{\sqrt{8M^3}}\int_{B_2} h_{\mu\nu}(x,L)T^{(2)}_{\mu\nu}\sqrt{-det\gamma_{\mu\nu}(L)}dx, \quad (51)$$

where $T^{(1)}_{\mu\nu}$ and $T^{(2)}_{\mu\nu}$ are energy-momentum tensors on brane 1 and 2, respectively.

In what follows we restrict ourselves to considering the fields on the brane at $y = L$ only, which we assume to be "our" brane. Substituting decompositions (41), (48) into (51) we find that the interaction of the tensor and the scalar fields with matter on the brane at $y = L$ in the Galilean coordinates is

$$\frac{1}{\sqrt{8M^3}}\int_{B_2}\left(\psi_0(L)b^0_{\mu\nu}(x)T^{\mu\nu} + \sum_{n=1}^\infty\psi_n(L)b^n_{\mu\nu}(x)T^{\mu\nu} - \frac{1}{2}\sum_{n=1}^\infty g_n(L)\varphi_n(x)T^\mu_\mu\right)dx. \qquad (52)$$

Thus, the couplings are defined by the values of the wave functions on the brane. The latter can be found only if we specify the background solutions $A(y)$ and $\phi(y)$.

4 Specific example

The considerations in the previous sections allowed us to find the general structure of the brane world models stabilized by the scalar field. To make predictions for the masses and the coupling constants we must take specific potentials for the scalar field and a particular vacuum solution of system (6).

To find an analytic solution to this system we will use the results of papers [5, 11]. Let us consider a special class of potentials, which can be represented as

$$V(\phi) = \frac{1}{8}\left(\frac{\delta W}{\delta \phi}\right)^2 - \frac{1}{24M^3}W^2(\phi).$$

It is easy to check that if we put

$$\phi'(y) = sign(y)\frac{1}{2}\frac{\delta W}{\delta \phi}, \quad A'(y) = sign(y)\frac{1}{24M^3}W(\phi), \tag{53}$$

then equations (6) are valid everywhere, except for the branes. In order the equations of motion be valid everywhere, one needs to finetune the brane potentials $\lambda_{1,2}(\phi)$.

Let us take $W(\phi)$ to be

$$W = 24M^3k - u\phi^2, \tag{54}$$

so that $V(\phi)$ is a quartic potential. Finetuned potentials on the branes can be chosen as follows:

$$\lambda_1 = W(\phi_1) + W'(\phi_1)(\phi - \phi_1) + \beta_1^2(\phi - \phi_1)^2, \tag{55}$$

$$\lambda_2 = -W(\phi_2) - W'(\phi_2)(\phi - \phi_2) + \beta_2^2(\phi - \phi_2)^2. \tag{56}$$

The parameters of the potentials $k, u, \phi_{1,2}, \beta_{1,2}$, when made dimensionless by the fundamental five-dimensional energy scale of the theory M, should be positive quantities of the order $O(1)$, i.e. there should be no hierarchical difference in the parameters.

For such a choice of the potentials the solution of the equations of motion is given by [5]

$$\phi(y) = \phi_1\, e^{-u|y|} \tag{57}$$

$$A(y) = k|y| + \frac{\phi_1^2}{48\,M^3}e^{-2u|y|}. \tag{58}$$

The interbrane distance is defined by the boundary conditions for the field ϕ and is expressed in terms of the parameters of the model by the relation

$$L = \frac{1}{u}\ln\left(\frac{\phi_1}{\phi_2}\right). \tag{59}$$

Thus, we see that the brane separation distance is stabilized.

Let us study the mass spectrum of tensor particles, which is defined by equation (35). The zero mode solution for arbitrary A is given by (40). For our choice of A it is impossible to find exact solutions for other modes. Therefore, in what follows we will use an approximation

$uL \ll 1$, which is rather general and physically interesting [6]. Keeping in A only the terms linear in y, we get

$$A(y) = \tilde{k}|y|, \quad \tilde{k} = k - \frac{\phi_1^2}{24M^3}u. \tag{60}$$

Thus, in this approximation equation (29) for the tensor field coincides with the equation of the unstabilized model, where a substitution $k \to \tilde{k}$ was made. This equation can be solved exactly, and the formulas for eigenfunctions and eigenvalues were discussed in detail in [8]. In particular, in this approximation the normalized functions ψ_0 for $m_0 = 0$ look like

$$\psi_0(y) = N_0 e^{-2\tilde{k}|y|}, \qquad N_0 = \frac{\tilde{k}^{\frac{1}{2}}}{(1 - e^{-2\tilde{k}L})^{\frac{1}{2}}} \tag{61}$$

in the coordinates, which are Galilean on the brane at $y = 0$, and

$$\psi_0(y) = N_0 e^{-2\tilde{k}|y|+2\tilde{k}L}, \qquad N_0 = \frac{\tilde{k}^{\frac{1}{2}}}{(e^{2\tilde{k}L} - 1)^{\frac{1}{2}}} \tag{62}$$

in the coordinates, which are Galilean on the brane at $y = L$. Formulas (37), (38), (39) give the following mass spectra

$$m_n^2 = \pi^2 \tilde{k}^2 n^2 e^{-2\tilde{k}L} \quad \text{(the brane at} \quad y = 0), \quad m_n^2 = \pi^2 \tilde{k}^2 n^2 \quad \text{(the brane at} \quad y = L). \tag{63}$$

Let us study now the mass spectra of the scalar particles. Equation and boundary conditions (42) - (44) for our choice of the potentials and background solutions take the form:

$$g_n'' + 2A'g_n' - 2\tfrac{\phi''}{\phi'}g_n' - \tfrac{(\phi')^2}{6M^3}g_n = -\mu_n^2 e^{2A} g_n, \tag{64}$$

$$(\beta_1^2 + u)g_n' + \mu_n^2 e^{2A} g_n|_{y=+0} = 0, \tag{65}$$

$$(\beta_2^2 - u)g_n' - \mu_n^2 e^{2A} g_n|_{y=L-0} = 0, \tag{66}$$

To find explicitly the eigenfunctions and eigenvalues of the problem we will use the same approximation $uL \ll 1$, which now implies

$$uy = uL\frac{y}{L} < uL \ll 1.$$

Substituting the explicit form of ϕ into (64) we get

$$g_n'' + 2A'g_n' + 2ug' - \frac{\phi_1^2}{6M^3}u^2 g_n + \mu_n^2 e^{2A} g_n = 0. \tag{67}$$

We solve this equation in the coordinates, which are Galilean on the brane at $y = L$. In this case $A(y)$ is expressed in terms of $\tilde{k}$ (60) as:

$$A(y) = \tilde{k}(y - L).$$

We have already found that for $\beta_2^2 - u > 0$ all eigenvalues of this problem are larger than zero. Therefore, we introduce a new variable by the relation

$$z = \frac{\mu_n}{\tilde{k}} e^{\tilde{k}y - \tilde{k}L}, \quad \frac{\mu_n}{\tilde{k}} e^{-\tilde{k}L} \le z \le \frac{\mu_n}{\tilde{k}}.$$

In terms of this variable the equation takes the form:

$$\frac{d^2 g_n}{dz^2} + \left(3 + \frac{2u}{\tilde{k}}\right)\frac{1}{z}\frac{dg_n}{dz} + \left(1 - \frac{b^2}{z^2}\right) g_n = 0, \quad b^2 = \frac{\phi_1^2}{6M^3}\frac{u^2}{\tilde{k}^2}. \tag{68}$$

Let us look for g_n in the form $g_n(z) = z^a t_n(z)$. Then the equation for t_n is

$$\frac{d^2 t_n}{dz^2} + \left(2a + 3 + \frac{2u}{\tilde{k}}\right)\frac{1}{z}\frac{dt_n}{dz} + \frac{a\left(a + 2 + \frac{2u}{\tilde{k}}\right)}{z^2} t_n + \left(1 - \frac{b^2}{z^2}\right) t_n = 0. \tag{69}$$

In order to turn this equation into the Bessel equation we put $a = -\left(1 + \frac{u}{\tilde{k}}\right)$ and get

$$\frac{d^2 t_n}{dz^2} + \frac{1}{z}\frac{dt_n}{dz} + \left(1 - \frac{\alpha^2}{z^2}\right) t_n = 0, \quad \alpha^2 = a^2 + b^2. \tag{70}$$

The general solution to this equation is

$$t_n(z) = AJ_\alpha(z) + BJ_{-\alpha}(z), \quad \alpha = \sqrt{a^2 + b^2}.$$

Correspondingly, we get

$$g_n(z) = z^{-\left(1+\frac{u}{\tilde{k}}\right)}\left(AJ_\alpha(z) + BJ_{-\alpha}(z)\right). \tag{71}$$

The boundary conditions in terms of z look like:

$$\begin{gathered} \tilde{k} z g_n + (\beta_1^2 + u)\frac{dg_n}{dz}\Big|_{z_1 = \frac{\mu_n}{\tilde{k}} e^{-\tilde{k}L}} = 0, \\ \tilde{k} z g_n - (\beta_2^2 - u)\frac{dg_n}{dz}\Big|_{z_2 = \frac{\mu_n}{\tilde{k}}} = 0. \end{gathered} \tag{72}$$

Below we show that for reproducing the Newtonian gravity on the brane at $y = L$ for strong five-dimensional gravity we must take $\tilde{k}L \sim 35$. In this case $z_1 = \frac{\mu_n}{\tilde{k}} e^{-\tilde{k}L} \approx 0$ is a very good approximation, and the boundary condition at zero allows us to drop the singular term with $J_{-\alpha}(z)$ in $g_n(z)$ since $B/A \sim e^{-2\tilde{k}L}$, and the corrections due to this term are negligible. Thus, up to normalization $g_n(z)$ can be written as

$$g_n(z) = z^{-\left(1+\frac{u}{\tilde{k}}\right)} J_\alpha(z),$$

The second boundary condition at $z_2 = \frac{\mu_n}{\tilde{k}}$ gives an equation for the mass spectrum of the scalar particles:

$$\left(1 + \alpha + \frac{u}{\tilde{k}} + \frac{\tilde{k} z_2^2}{\beta_2^2 - u}\right) J_\alpha(z_2) - z_2 J_{\alpha-1}(z_2) = 0. \tag{73}$$

Expanding the Bessel function for small z_2 up to the second term and keeping the terms up to the order z_2^2 in the equation, we get the following relation for the mass of the lowest scalar excitation:

$$\mu_1^2 = \frac{4\tilde{k}^2(-1 + \alpha - \frac{u}{\tilde{k}})(1 + \alpha)(\beta_2^2 - u)}{4\tilde{k}(1 + \alpha) + (1 + \alpha - \frac{u}{\tilde{k}})(\beta_2^2 - u)}. \tag{74}$$

For small $u/\tilde{k}$ it reduces to

$$\mu_1^2 = \frac{\phi_1^2 u^2}{3M^3} \frac{\beta_2^2 - u}{\beta_2^2 + 4\tilde{k}}, \tag{75}$$

which for $\beta_2^2 \to \infty$ coincides with the results of papers [6, 7] in our approximation. The next roots of equation (73) are of the order $\tilde{k}^2$, and the asymptotic formula of the general theory [9] for large n gives $\mu_n^2 = \pi^2 \tilde{k}^2 n^2$.

The normalization condition for the eigenfunctions $\{g_n(y), n = 0, 1, ...\}$ of the problem (42) - (44), which gives the canonical kinetic terms in (50), is the following:

$$\frac{3}{2} \int_0^L dy e^{2A} \left(g_n g_k + \frac{6M^3}{(\phi')^2} g_n' g_k' \right) = \delta_{nk}. \tag{76}$$

The normalized functions $g_n(y)$ look like

$$g_n(y) = A_n \left(\frac{\mu_n}{\tilde{k}} e^{\tilde{k}y - \tilde{k}L} \right)^{-\left(1 + \frac{u}{\tilde{k}}\right)} J_\alpha \left(\frac{\mu_n}{\tilde{k}} e^{\tilde{k}y - \tilde{k}L} \right), \tag{77}$$

$$A_n = \frac{u\hat{\kappa}\phi_1 \left(\mu_n/\tilde{k}\right)^{1+u/\tilde{k}}}{M^{\frac{3}{2}} 3\mu_n} \left[\frac{\sqrt{\pi} \left(\mu_n/\tilde{k}\right)^{\alpha-1} H(-\alpha, 2\mu_n/\tilde{k}) \Gamma(-\alpha + 3/2)}{2^{2(\alpha+1)} \tilde{k} \Gamma(2+\alpha)\Gamma(1+\alpha)} + \frac{J_\alpha^2 \left(\mu_n/\tilde{k}\right)}{\beta_2^2 - u} \right]^{-\frac{1}{2}}.$$

Here $H(a, y)$ is the Struve function. We also note that in our approximation the boundary term at $y = 0$ is dropped in the normalization condition.

Now that we have found explicit expressions for the wave functions of the tensor and the scalar fields, we can find their coupling constants to matter on brane 2.

The coefficient in front of the zero mode of the tensor field $b_{\mu\nu}^0(x)$ in (51) can be expressed in terms of the Planck mass on the brane at $y = L$ as $1/\sqrt{8M_{Pl}^2}$. Then equation (62) implies:

$$\frac{1}{\sqrt{8M_{Pl}^2}} = \frac{1}{\sqrt{8M^3}} N_0 = \frac{1}{\sqrt{8M^3}} \frac{\tilde{k}^{\frac{1}{2}}}{(e^{2\tilde{k}L} - 1)^{\frac{1}{2}}}, \tag{78}$$

which gives a relation between the Planck mass on the negative tension brane and the fundamental five-dimensional energy scale M

$$M_{Pl}^2 = M^3 \frac{e^{2\tilde{k}L} - 1}{\tilde{k}}. \tag{79}$$

Again we have obtained the formulas, which coincide with the formulas of the unstabilized model after replacing $k \to \tilde{k}$. Therefore, in order to get the correct value of the gravitational constant on the brane at $y = L$ on can take the same values of the parameters, which are used in the unstabilized model, i.e. $M \sim \tilde{k} \sim 1TeV, \quad \tilde{k}L \sim 35$ [6]. Then the parameter u may be of the order of ten GeV, and the radion mass (75) may be of the order of hundred GeV.

The exact solution for the zero mode of the tensor field (40) allows one to get the relation between the Planck mass on the brane at $y = L$ and the fundamental five-dimensional energy

scale beyond the approximation used here. Other possible approximations were studied in paper [6].

Formula (52) shows that the coupling constant of the n-th scalar mode to matter is defined by the value of its wave function on the brane and is approximately given by the relation

$$\epsilon_n = -\frac{A_n}{\sqrt{32M^3}} J_\alpha \left(\frac{\mu_n}{\tilde{k}}\right) \left(\frac{\mu_n}{\tilde{k}}\right)^{-(1+u/\tilde{k})} .$$

For the radion, this constant is

$$\epsilon_1 \simeq -\frac{\mu_1}{\sqrt{24M^3\tilde{k}}} \sqrt{\frac{\beta_2^2 + 4\tilde{k}^2}{\beta_2^2 - u}},$$

and turns out to be of the order $\epsilon_1^{-1} \sim 50TeV$ for the above given values of the model parameters.

5 Discussion

In the present paper we have considered the general structure of the brane world models stabilized by the scalar field. For an arbitrary background configuration of the gravitational and scalar fields, satisfying the equations of motion for a stabilized model, we constructed the second variation Lagrangian and derived the equations for the fields describing the fluctuations against the background. A convenient gauge and a substitution were found, which enabled us to decouple the equations of motions and to isolate the five-dimensional degrees of freedom. It was shown that the tensor sector splits from the scalar one and has the same structure, as in the unstabilized model. Namely, for any background there is a massless four-dimensional graviton and a tower of massive tensor fields, which represent the four-dimensional degrees of freedom in the tensor sector.

The structure of the scalar sector was found to be more complicated: the operator of the mass squared turned out to be non-self-adjoint and its eigenfunctions to be non-orthogonal. Nevertheless, the structure of the five-dimensional action is such that the non-diagonal interaction terms of the four-dimensional scalar modes vanish.

For a particular choice of the background solution and the parameters of the model it was found that the influence of the scalar field background on the tensor excitations reduces to a renormalization of the parameter k of the unstabilized model, which is replaced by $\tilde{k}$ (60). In this case the inverse size of extra dimension and the masses of tensor excitations may be of the order of $1TeV$, the radion mass being of the order of $100GeV$. For this choice of the parameters the radion coupling constant to matter on the brane at $y = L$ turned out to be approximately ten times smaller, than for the same choice of the parameters in the unstabilized model.

This result does not mean that the radion coupling to matter on the negative tension brane is always weak in the stabilized models. It is quite possible that it can be much larger for a different choice of the model parameters. In this case the radion mass and the masses of the tensor excitations must also shift. To find out, whether it is really so, one has to scan the

whole parameter space of the model. The coupling of the radion to matter can also become stronger due to the radion-Higgs mixing as discussed, for example, in [7, 12, 13, 14].

Acknowledgments

The work is partly supported by RFBR 04-02-16476, RFBR 04-02-17448, Universities of Russia UR.02.02.503, and Russian Ministry of Education and Science NS.1685.2003.2 grants. The authors are grateful to Yu. Grats, P. Ermolov, Yu. Kubyshin and V. Rubakov for useful discussions. I.V. would also like to thank A. Belyaev, S. Chivukula and C.-P. Yuan for interesting discussions.

Список литературы

[1] F. Feruglio, *Eur. Phys. J. C* **33** (2004) S114.
S. Mele, *Eur. Phys. J. C* **33** (2004) S919.
I. Antoniadis, *Eur. Phys. J. C* **33** (2004) S914.
C. Csaki, "TASI lectures on extra dimensions and branes", arXiv:hep-ph/0404096.
J. Hewett, J. March-Russell, *Phys. Lett. B* **592** (2004) 1.

[2] V.A. Rubakov, *Phys. Usp.* **44** (2001) 871 [arXiv:hep-ph/0104152].

[3] L. Randall, R. Sundrum, *Phys. Rev. Lett.* **83** (1999) 3370.

[4] W.D. Goldberger, M.B. Wise, *Phys. Rev. Lett.* **83** (1999) 4922.

[5] O. DeWolfe, D.Z. Freedman, S.S. Gubser, A. Karch, *Phys. Rev. D* **62** (2000) 046008.

[6] E.E. Boos, Yu.S. Mikhailov, M.N. Smolyakov, I.P. Volobuev, *Nucl. Phys. B* **717** (2005) 19.

[7] C. Csaki, M.L. Graesser, G.D. Kribs, *Phys. Rev. D* **63** (2001) 065002.

[8] E.E. Boos, I.P. Volobuev, Yu.A. Kubyshin, M.N. Smolyakov, *Theor. Math. Phys.* **131** (2002) 629 [arXiv:hep-th/0105304]; *Class. Quant. Grav.* **19** (2002) 4591.

[9] V.M. Babich, M.B. Kapilevich, S.G. Mikhlin, "Linear Equations of Mathematical Physics", Ch. II, §3 (in Russian), Nauka, Moscow, 1964.

[10] L.D. Landau, E.M. Lifshitz, "The Classical Theory of Fields", Pergamon Press, Oxford (1975).

[11] A. Brandhuber, K. Sfetsos, *JHEP* **9910** (1999) 013.

[12] G.F. Giudice, R. Rattazzi, J.D. Wells, *Nucl. Phys. B* **595** (2001) 250.

[13] T. Han, G.D. Kribs, B. McElrath, *Phys. Rev. D* **64**, (2001) 076003.

[14] J.L. Hewett, T.G. Rizzo, *JHEP* **0308** (2003) 028.

Эдуард Эрнстович Боос
Игорь Павлович Волобуев
Юрий Сергеевич Михайлов
Михаил Николаевич Смоляков

Физические степени свободы в стабилизированных моделях мира на бране

Препринт НИИЯФ МГУ 2005-30/796

Работа поступила в ОНТИ 05.12.2005 г.