New Relations between QCD Sum Rules for $g_{\eta\Sigma\Sigma}$ and $g_{\eta\Lambda\Lambda}$ Coupling Constants

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Summary

New relations between QCD Borel sum rules for strong coupling constants are derived. It is shown that starting from the sum rule for the coupling constant $g_{\eta\Sigma\Sigma}$ it is straightforward to obtain the corresponding sum rule for the $g_{\eta\Lambda\Lambda}$ *et vice versa.* The ratio $g_{\eta\Sigma\Sigma}/g_{\eta\Lambda\Lambda} = -0.66$ is calculated and the value of $g_{\eta\Lambda\Lambda}$ equal to -3.39 is deduced

Резюме

Получены новые соотношения между борелевскими правилами сумм в КХД для масс Σ^0 и Λ гиперонов. Показано, что, отправляясь от правила сумм для $g_{\eta\Sigma\Sigma}$, можно непосредственно получить соответствующее правило сумм для $g_{\eta\Lambda\Lambda}$ *et vice versa.* Получено отношение $g_{\eta\Sigma\Sigma}/g_{\eta\Lambda\Lambda} = -0.66$ и определено значение константы $g_{\eta\Lambda\Lambda}$ равное -3.39.

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1 Introduction

QCD sum rules has proved to be a powerful tool for studying the properties of the low-lying hadrons [1]. Many attention was paid to magnetic moments [2, 3, 4, 5, 6] and axial-vector coupling constants in the framework of the Borel sum rules [7, ?]. Analysis of the meson-baryon couplings in the framework of the QCD Borel sum rules are also of great interest as these couplings enter many physical problems [8, 9, 10, 11, 12]. Recently we have found intercrossed relations between QCD sum rules for Σ^0 and Λ hyperons constructed for such important characteristics as masses and magnetic moments [14], [15], [16].

It is natural to put a question whether similar relations can be constructed for other quantities such as pseudoscalar meson-baryon coupling constants. The problem has also a practical side, as, for example, QCD sum rules in [11, 12] were written for all $g_{\pi BB}$ and $g_{\eta BB}$ coupling constants but the $g_{\eta\Lambda\Lambda}$ one.

We shall show here in what way one can construct QCD sum rule for the $g_{\eta\Lambda\Lambda}$ coupling constant on example of the known QCD sum rules derived in [11, 12].

2 Nonlinear relation between $g_{\eta\Lambda\Lambda}$ and $g_{\eta\Sigma^0\Sigma^0}$ in the SU(3)

We begin as in [14, 15] from a simple example. In the unitary model all the pseudoscalar meson-baryon coupling constants can be expressed in terms of F and D constants from the known unitary symmetry Lagrangian [13]

$$L = DSp\bar{B}\{P, B\} + FSp\bar{B}[P, B]. \tag{1}$$

wherefrom

$$g_{\pi NN} = F + D, \quad g_{\pi\Sigma\Sigma} = -\sqrt{2}F, \quad g_{\pi\Xi\Xi} = -F + D, \quad g_{\pi\Sigma\Lambda} = -\sqrt{\frac{2}{3}}D,$$
$$g_{\eta NN} = \frac{1}{\sqrt{6}}(3F - D), \quad g_{\eta\Sigma\Sigma} = \sqrt{\frac{2}{3}}D,$$
$$g_{\eta\Xi\Xi} = -\frac{1}{\sqrt{6}}(3F + D), \quad g(\eta\Lambda\Lambda) = -\sqrt{\frac{2}{3}}D. \tag{2}$$

But these coupling constants for π and η mesons can be written in a form similar to that found for Σ-like baryon magnetic moments in $SU(3)_f$ [17]:

$$\mu(B(qq', h) = (e_q + e_{q'})\mu_F + e_h(\mu_F - \mu_D),$$

wherefrom usual unitary symmetry pattern for the magnetic moments of the Σ-like baryon emerges:

$$\mu(p) = \mu(\Sigma^+) = \mu_F + \frac{1}{3}\mu_D,$$
$$\mu(\Xi^-) = \mu(\Sigma^-) = -\mu_F + \frac{1}{3}\mu_D,$$
$$\mu(n) = \mu(\Xi^0) = -\frac{2}{3}\mu_D.$$

Namely, let us write coupling constants of π^0 and η mesons related in the quark model to currents

$$j^{\pi^0} = \frac{1}{\sqrt{2}}[\bar{u}\gamma_5 u - \bar{d}\gamma_5 d] \tag{3}$$

and

$$j^{\eta} = \frac{1}{\sqrt{6}}[\bar{u}\gamma_5 u + \bar{d}\gamma_5 d - 2\bar{s}\gamma_5 s]. \tag{4}$$

with octet Σ-like baryons as $B(qq,h)$

$$g_{\mathcal{M}B(qq,h)B(qq,h)} = g_{\mathcal{M}qq}2F + g_{\mathcal{M}hh}(F-D), \tag{5}$$

or, particle per particle:

$$g_{\pi^0 pp} = g_{\pi uu}2F + g_{\pi dd}(F-D) = \sqrt{\frac{1}{2}}(F+D);$$

$$g_{\pi^0\Sigma^+\Sigma^+} = g_{\pi uu}2F + g_{\pi ss}(F-D) = \sqrt{2}F;$$

$$g_{\pi^0\Xi^0\Xi^0} = g_{\pi ss}2F + g_{\pi uu}(F-D) = \sqrt{\frac{1}{2}}(F-D);$$

and so on, where $g_{\pi uu} = +\sqrt{\frac{1}{2}}$, $g_{\pi dd} = -\sqrt{\frac{1}{2}}$ and $g_{\pi ss} = 0$ can be just read off Eq.(3); and

$$g_{\eta pp} = g_{\eta uu}2F + g_{\eta dd}(F-D) = \sqrt{\frac{1}{6}}(3F-D);$$

$$g_{\eta\Sigma^+\Sigma^+} = g_{\eta uu}2F + g_{\eta ss}(F-D) = \sqrt{\frac{2}{3}}D;$$

$$g_{\eta\Xi^0\Xi^0} = g_{\eta ss}2F + g_{\eta uu}(F-D) = -\sqrt{\frac{1}{6}}(3F+D);$$

and so on, where $g_{\eta uu} = +\sqrt{\frac{1}{6}}$, $g_{\eta dd} = +\sqrt{\frac{1}{6}}$ and $g_{\eta ss} = -\sqrt{\frac{2}{3}}$ are read off the Eq.(4).

The $SU(3)$ summetry model gives for

$$g_{\eta\Lambda\Lambda} = -\sqrt{\frac{2}{3}}D \tag{6}$$

Let us write now the relation for $g_{\eta\Sigma^0\Sigma^0}$ coupling constant:

$$g_{\eta\Sigma^0\Sigma^0} = g_{\eta uu}F + g_{\eta dd}F + g_{\eta ss}(F-D) = \sqrt{\frac{2}{3}}D, \tag{7}$$

and change $(d \leftrightarrow s)$ to form an auxiliary quantity

$$g_{\eta\tilde{\Sigma}^0_{ds}\tilde{\Sigma}^0_{ds}} = g_{\eta uu}F + g_{\eta ss}F + g_{\eta dd}(F-D) = -\sqrt{\frac{1}{6}}D, \tag{8}$$

and $(u \leftrightarrow s)$ to form one more auxiliary quantity

$$g_{\eta\tilde{\Sigma}^0_{us}\tilde{\Sigma}^0_{us}} = g_{\eta dd}F + g_{\eta ss}F + g_{\eta uu}(F-D) = -\sqrt{\frac{1}{6}}D. \tag{9}$$

The following relation holds:

$$2g_{\eta\tilde{\Sigma}^0_{ds}\tilde{\Sigma}^0_{ds}} + 2g_{\eta\tilde{\Sigma}^0_{us}\tilde{\Sigma}^0_{us}} - g_{\eta\Sigma^0\Sigma^0} = 3g_{\eta\Lambda\Lambda}. \tag{10}$$

The origin of this relation lies in the structure of baryon wave functions in the NRQM with isospin $I = 1, 0$ and $I_3 = 0$:

$$2\sqrt{3}|\Sigma^0(ud,s)\rangle_\uparrow = |2u_\uparrow d_\uparrow s_\downarrow + 2d_\uparrow u_\uparrow s_\downarrow - u_\uparrow s_\uparrow d_\downarrow - s_\uparrow u_\uparrow d_\downarrow - d_\uparrow s_\uparrow u_\downarrow - s_\uparrow d_\uparrow u_\downarrow\rangle,$$

$$2|\Lambda\rangle_\uparrow = |d_\uparrow s_\uparrow u_\downarrow + s_\uparrow d_\uparrow u_\downarrow - u_\uparrow s_\uparrow d_\downarrow - s_\uparrow u_\uparrow d_\downarrow\rangle,$$

where $q_\uparrow$ ($q_\downarrow$) means wave function of the quark q (here $q = u, d, s$) with the helicity +1/2 (-1/2). With the exchanges $d \leftrightarrow s$ and $u \leftrightarrow s$ one arrives at the corresponding U-spin and V-spin quantities, so $U = 1, 0$ and $U_3 = 0$ baryon wave functions are

$$-2|\tilde{\Sigma}^0_{d\leftrightarrow s}(us,d)\rangle = |\Sigma^0(ud,s) > +\sqrt{3}|\Lambda\rangle,$$

$$-2|\tilde{\Lambda}_{d\leftrightarrow s}\rangle = -\sqrt{3}|\Sigma^0(ud,s) > +|\Lambda\rangle,$$

while $V = 1, V_3 = 0$ and $V = 0$ baryon wave functions are

$$-2|\tilde{\Sigma}^0_{u\leftrightarrow s}(ds,u)\rangle = |\Sigma^0(ud,s)\rangle - \sqrt{3}|\Lambda\rangle,$$

$$2|\tilde{\Lambda}_{u\leftrightarrow s}\rangle = \sqrt{3}|\Sigma^0(ud,s)\rangle + |\Lambda\rangle.$$

It is easy to show that the relation given by Eq.(10) immeaditely follows.

3 Relation between QCD correlators for Σ^0 and Λ hyperons

Now we demonstrate how similar considerations work for QCD sum rules on the example of QCD Borel sum rules for pseudoscalar meson-baryon coupling constants.

The starting point would be two-point Green's function for hyperons Σ^0 and Λ of the baryon octet:

$$\Pi^{\Sigma^0,\Lambda} = i\int d^4x e^{ipx} < 0|T\{J^{\Sigma^0,\Lambda}(x), J^{\Sigma^0,\Lambda}(0)\}|\eta >, \tag{11}$$

where isovector (with $I_3 = 0$) and isocalar field operators could be chosen as [12]

$$J^{\Sigma^0} = \frac{1}{2}\epsilon_{abc}[(\mathrm{u^{aT}Cs^b})\gamma_5\mathrm{d^c} - (\mathrm{d^{aT}Cs^b})\gamma_5\mathrm{u^c} - (\mathrm{u^{aT}C\gamma_5 s^b})\mathrm{d^c} + (\mathrm{d^{aT}C\gamma_5 s^b})\mathrm{u^c}],$$

$$J^\Lambda = \frac{1}{2\sqrt{3}}\epsilon_{abc}[-2(\mathrm{u^{aT}Cd^b})\gamma_5\mathrm{s^c} + (\mathrm{u^{aT}Cs^b})\gamma_5\mathrm{d^c} + (\mathrm{d^{aT}Cs^b})\gamma_5\mathrm{u^c} + 2(\mathrm{u^{aT}C\gamma_5 d^b})\mathrm{s^c} - (\mathrm{u^{aT}C\gamma_5 s^b})\mathrm{d^c} - (\mathrm{d^{aT}C\gamma_5 s^b})\mathrm{u^c}], \tag{12}$$

where a, b, c are the color indices and u, d, s are quark wave functions, C is charge conjugation matrix,

We show now that one can operate with with η - coupling to Σ hyperon and obtain the results for the η- coupling to the Λ hyperon. The reasoning would be valid also for charm and beaty Σ-like and Λ-like baryons.

In order to arrive at the desired relations we write not only isospin quantities but also U-spin and V-spin ones.

Let us introduce U-vector (with $U_3 = 0$) and U-scalar field operators just formally changing $(d \leftrightarrow s)$ in the Eq.(12):

$$\begin{aligned}
\tilde{J}^{\Sigma^0(d\leftrightarrow s)} &= \frac{1}{2}\epsilon_{abc}[(\mathrm{u^{aT}C \cdot 1 \cdot d^b})\gamma_5 \mathrm{s^c} - (\mathrm{s^{aT}Cd^b}) \cdot \gamma_5 \cdot \mathrm{u^c} - (1 \leftrightarrow \gamma_5)] \\
\tilde{J}^{\Lambda(d\leftrightarrow s)} &= \frac{1}{2\sqrt{3}}\epsilon_{abc}[(-2(\mathrm{u^{aT}C \cdot 1 \cdot s^b})\gamma_5 \mathrm{d^c} + (\mathrm{u^{aT}C \cdot 1 \cdot d^b}) \cdot \gamma_5 \cdot \mathrm{s^c} + \\
&\qquad (\mathrm{s^{aT}C \cdot 1 \cdot d^b}) \cdot \gamma_5 \cdot \mathrm{u^c}) - (1 \leftrightarrow \gamma_5)]. \qquad (13)
\end{aligned}$$

Similarly we introduce V-vector (with $V_3 = 0$) and V-scalar field operators just changing $(u \leftrightarrow s)$ in the Eq.(12):

$$\begin{aligned}
\tilde{J}^{\Sigma^0(u\leftrightarrow s)} &= \frac{1}{2}\epsilon_{abc}[(\mathrm{s^{aT}C \cdot 1 \cdot u^b})\gamma_5 \mathrm{d^c} - (\mathrm{d^{aT}Cu^b}) \cdot \gamma_5 \cdot \mathrm{s^c} - (1 \leftrightarrow \gamma_5)] \\
\tilde{J}^{\Lambda(u\leftrightarrow s)} &= \frac{1}{2\sqrt{3}}\epsilon_{abc}[(-2(\mathrm{u^{aT}C \cdot 1 \cdot s^b})\gamma_5 \mathrm{u^c} + (\mathrm{u^{aT}C \cdot 1 \cdot u^b}) \cdot \gamma_5 \cdot \mathrm{s^c} + \\
&\qquad (\mathrm{d^{aT}C \cdot 1 \cdot u^b}) \cdot \gamma_5 \cdot \mathrm{s^c}) - (1 \leftrightarrow \gamma_5)]. \qquad (14)
\end{aligned}$$

Field operators of the Eq.(12) and Eq.(13) can be related through

$$\begin{aligned}
-2\tilde{J}^{\Lambda(d\leftrightarrow s)} &= \sqrt{3}J^{\Sigma^0} + J^{\Lambda}, \\
2\tilde{J}^{\Sigma^0(d\leftrightarrow s)} &= J^{\Sigma^0} - \sqrt{3}J^{\Lambda}, \\
2\tilde{J}^{\Lambda(u\leftrightarrow s)} &= \sqrt{3}J^{\Sigma^0} + J^{\Lambda}, \\
2\tilde{J}^{\Sigma^0(u\leftrightarrow s)} &= J^{\Sigma^0} + \sqrt{3}J^{\Lambda}. \qquad (15)
\end{aligned}$$

Upon using Eqs.(12-15) two-point functions of the Eq.(11) for hyperons Σ^0 and Λ of the baryon octet can be related as

$$2[\tilde{\Pi}^{\Sigma^0(d\leftrightarrow s)} + \tilde{\Pi}^{\Sigma^0(u\leftrightarrow s)}] - \Pi^{\Sigma^0} = 3\Pi^{\Lambda}, \qquad (16)$$

$$2[\tilde{\Pi}^{\Lambda(d\leftrightarrow s)} + \Pi^{\tilde{\Lambda}(u\leftrightarrow s)}] - \Pi^{\Lambda} = 3\Pi^{\Sigma^0}. \qquad (17)$$

In [12] the pion and eta-coupling constants to octet baryons were calculated within the framework of the Light-Cone QCD. It was shown that the corresponding LC QCD SR's respected the unitary symmetry pattern.

We rewrite LC QCD SR's (Eq.(40) from [12]), taking only those with the η-meson, in a way to make clear unitary symmetry pattern:

$$\begin{aligned}
-\sqrt{3}m_N\lambda_N^2 g_{\eta NN}e^{-(M^2/m_N^2)} &= \Pi_1^\gamma(M^2) - \Pi_2^\gamma(M^2) = \\
&\qquad g_{\eta qq}\Pi_1^\gamma + g_{\eta q'q'}(-\Pi_2^\gamma), \quad q, q' = u, d; \\
-\sqrt{3}m_\Sigma\lambda_\Sigma^2 g_{\eta\Sigma\Sigma}e^{-(M^2/m_\Sigma^2)} &= \Pi_1^\gamma(M^2) + 2\Pi_2^\gamma(M^2) = \\
&\qquad g_{\eta qq}\Pi_1^\gamma + g_{\eta ss}(-\Pi_2^\gamma); \\
-\sqrt{3}m_\Xi\lambda_\Xi^2 g_{\eta\Xi\Xi}e^{-(M^2/m_\Xi^2)} &= -2\Pi_1^\gamma(M^2) - \Pi_2^\gamma(M^2) = \\
&\qquad g_{\eta ss}\Pi_1^\gamma + g_{\eta qq}(-\Pi_2^\gamma); \qquad (18)
\end{aligned}$$

where $\Pi^{\gamma}_{1,2}(M^2)$ are given in [12] and are rather cumbersome. Here for us it is important that Π^{γ}_1 corresponds exactly to $2F$, that is η-meson interacts with a quark q from the biquark (qq) of the baryon $B(qq,q')$, while $(-\Pi^{\gamma}_2)$ corresponds exactly to $(F-D)$ of the Eq.(5), that is η-meson interacts with a quark q'.

Thus up to a renormalization factor $m_B\lambda^2_B e^{-(m^2_B/M^2)}$ relations Eq.(18) and Eq.(3) are identical that is QCD sum rules shows implicit unitary symmetry pattern. Following reasoning of the previous section and analogues of the Eq.(10) we obtain

$$-\sqrt{3}m_\Lambda\lambda^2_\Lambda g_{\eta\Lambda\Lambda}e^{-(M^2/m^2_\Lambda)} = -\Pi^{\gamma}_1(M^2) - 2\Pi^{\gamma}_2(M^2). \tag{19}$$

But in [12] LC QCD SR's are flavour symmetric So we search to investigate a more complicated case where unitary symmetry of v.e.v. and quark masses is broken.

Recently Kim et al.[11] have elaborated QCD SR for the pion- and η- coupling constants to octet baryons taking into account corrections due to m_s and $<\bar{s}s>$ so it would be ideal for us to take it as a probe and an independent test of our reasoning.

QCD Borel sum rules were obtained in [11] for coupling of the octet Σ-like baryons to π^0 and η. It is convenient for us to rewrite the result of [11] only for η-meson coupling to Σ^0 hyperon as

$$\begin{aligned}\frac{1}{\sqrt{2}}m^2_\eta\lambda^2_\Sigma g_{\eta\Sigma^0\Sigma^0}e^{-(M^2/m^2_\Sigma)}[1+A_\Sigma M^2] =\\ g_{\eta ss}m^2_\eta M^4E_0(x)[\frac{\langle\bar{s}s\rangle}{12\pi^2 f_\eta}+\frac{3f_{3\eta}}{4\sqrt{2}\pi^2}]\\ -g_{\eta ss}\frac{1}{f_\eta}M^2(m_d\langle\bar{u}u\rangle+m_u\langle\bar{d}d\rangle)\langle\bar{s}s\rangle\\ -g_{\eta ss}\frac{m^2_\eta}{72f_\eta}\langle\bar{s}s\rangle<\frac{\alpha_s}{\pi}\mathcal{G}^2>\\ +\frac{1}{6f_\eta}m^2_0[\langle\bar{s}s\rangle(m_d g_{\eta uu}\langle\bar{u}u\rangle+m_u g_{\eta dd}\langle\bar{d}d\rangle)\\ +m_s(g_{\eta uu}+g_{\eta dd})\langle\bar{u}u\rangle\langle\bar{d}d\rangle].\end{aligned} \tag{20}$$

where m_q, $q=u,d,s$ are current quark masses, f_η is a decay constant, taken as $f_\eta=1.2f_\pi$ in calculations, $<\bar{q}q>$, $q=u,d,s$ are v.e.v.'s, taken as $-(2\pi)^2<\bar{u}u>=-(2\pi)^2<\bar{d}d>=-(2\pi)^2<\bar{q}q>=0.55$ GeV^3, $<\bar{s}s>/<\bar{d}d>=0.8$, $m^2_0=0.8$ GeV^2. $E_n(x)$ are the factors used to subtract the continuum contribution, $x=W^2/M^2$. $W^2=2.0$ GeV^2 is taken with the overlap amplitude $(2\pi)^4\lambda^2_\Sigma=1.88$ GeV^6.

The η-quark coupling constants $g_{\eta ss}$, $q=u,d,s$ are read off the Eq.(4). Finally,

$$\begin{aligned}\sqrt{3}m^2_\eta\lambda^2_\Sigma g_{\eta\Sigma^0\Sigma^0}e^{-(M^2/m^2_\Sigma)}[1+A_\Sigma M^2] = -2m^2_\eta M^4E_0(x)[\frac{\langle\bar{s}s\rangle}{12\pi^2 f_\eta}+\frac{3f_{3\eta}}{4\sqrt{2}\pi^2}]\\ +\frac{2}{f_\eta}M^2(m_d\langle\bar{u}u\rangle+m_u\langle\bar{d}d\rangle)\langle\bar{s}s\rangle+\frac{2m^2_\eta}{72f_\eta}\langle\bar{s}s\rangle<\frac{\alpha_s}{\pi}\mathcal{G}^2>\\ +\frac{1}{6f_\eta}m^2_0[\langle\bar{s}s\rangle(m_d\langle\bar{u}u\rangle+m_u\langle\bar{d}d\rangle)+2m_s\langle\bar{u}u\rangle\langle\bar{d}d\rangle].\end{aligned} \tag{21}$$

Now we are able to construct two auxiliary sum rules for $g_{\eta\tilde{\Sigma}^0_{ds}\tilde{\Sigma}^0_{ds}}$ and $g_{\eta\tilde{\Sigma}^0_{us}\tilde{\Sigma}^0_{us}}$ upon changes $d\leftrightarrow s$ and $u\leftrightarrow s$ in the Eq.(20). Note that the η-quark current coefficients should be taken into account only in the quark lines with the attached η-line. So,

$$\sqrt{3}m_\eta^2\lambda^2_{\tilde{\Sigma}^0_{ds}}g_{\eta\tilde{\Sigma}^0_{ds}\tilde{\Sigma}^0_{ds}}e^{-(m^2_{\tilde{\Sigma}^0_{ds}}/M^2)}(1+A_{\tilde{\Sigma}^0_{ds}}M^2)=$$
$$m_\eta^2M^4E_0(x)[\frac{\langle\bar{d}d\rangle}{12\pi^2f_\eta}+\frac{3f_{3\eta}}{4\sqrt{2}\pi^2}]$$
$$-\frac{1}{f_\eta}M^2(m_u\langle\bar{s}s\rangle+m_s\langle\bar{u}u\rangle)\langle\bar{d}d\rangle$$
$$-\frac{m_\eta^2}{72f_\eta}\langle\bar{d}d\rangle<\frac{\alpha_s}{\pi}\mathcal{G}^2>+\frac{1}{6f_\eta}m_0^2[-2m_u\langle\bar{d}d\rangle\langle\bar{s}s\rangle$$
$$-m_d\langle\bar{u}u\rangle\langle\bar{s}s\rangle+m_s\langle\bar{u}u\rangle\langle\bar{d}d\rangle]; \quad (22)$$

$$\sqrt{3}m_\eta^2\lambda^2_{\tilde{\Sigma}^0_{us}}g_{\eta\tilde{\Sigma}^0_{us}\tilde{\Sigma}^0_{us}}e^{-(m^2_{\tilde{\Sigma}^0_{us}}/M^2)}(1+A_{\tilde{\Sigma}^0_{us}}M^2)=$$
$$m_\eta^2M^4E_0(x)[\frac{\langle\bar{u}u\rangle}{12\pi^2f_\eta}+\frac{3f_{3\eta}}{4\sqrt{2}\pi^2}]$$
$$-\frac{1}{f_\eta}M^2(m_u\langle\bar{s}s\rangle+m_s\langle\bar{u}u\rangle)\langle\bar{d}d\rangle$$
$$-\frac{m_\eta^2}{72f_\eta}\langle\bar{u}u\rangle<\frac{\alpha_s}{\pi}\mathcal{G}^2>+\frac{1}{6f_\eta}m_0^2[-2m_d\langle\bar{u}u\rangle\langle\bar{s}s\rangle$$
$$-m_u\langle\bar{d}d\rangle\langle\bar{s}s\rangle+m_s\langle\bar{u}u\rangle\langle\bar{d}d\rangle]. \quad (23)$$

Now we can construct sum rule for Λ coupling to η-meson. Upon using the relations (16,20-23) we get

$$\frac{1}{\sqrt{2}}m_\eta^2\lambda_\Lambda^2g_{\eta\Lambda\Lambda}e^{-(m_\Lambda^2/M^2)}[1+A_\Lambda M^2]=$$
$$\frac{m_\eta^2M^4E_0(x)}{36\pi^2f_\eta}[(2g_{\eta uu}\langle\bar{u}u\rangle+2g_{\eta dd}\langle\bar{d}d\rangle-g_{\eta ss}\langle\bar{s}s\rangle)+\frac{9f_\eta f_{3\eta}}{4\sqrt{2}\pi^2}]$$
$$-\frac{M^2}{3f_\eta}[[(2g_{\eta uu}-g_{\eta ss})m_d\langle\bar{u}u\rangle+(2g_{\eta dd}-g_{\eta ss})m_u\langle\bar{d}d\rangle]\langle\bar{s}s\rangle-$$
$$+2m_s(g_{\eta uu}+g_{\eta dd})\langle\bar{u}u\rangle\langle\bar{d}d\rangle]$$
$$-\frac{m_\eta^2}{216f_\eta}[2g_{\eta uu}\langle\bar{u}u\rangle+2g_{\eta dd}\langle\bar{d}d\rangle-g_{\eta ss}\langle\bar{s}s\rangle]<\frac{\alpha_s}{\pi}\mathcal{G}^2>$$
$$+\frac{m_0^2}{18f_\eta}[4g_{\eta ss}(m_d\langle\bar{u}u\rangle+m_u\langle\bar{d}d\rangle)\langle\bar{s}s\rangle+m_s(g_{\eta uu}+g_{\eta dd})\langle\bar{u}u\rangle\langle\bar{d}d\rangle+$$
$$(g_{\eta dd}m_ug_{\eta dd}\langle\bar{d}d\rangle+g_{\eta uu}m_d\langle\bar{u}u\rangle)\langle\bar{s}s\rangle]. \quad (24)$$

Finally,

$$\sqrt{3}m_\eta^2\lambda_\Lambda^2g_{\eta\Lambda\Lambda}e^{-(M^2/m_\Lambda^2)}[1+A_\Lambda M^2]=$$
$$\frac{2}{3}m_\eta^2M^4E_0(x)[\frac{\langle\bar{u}u\rangle+\langle\bar{d}d\rangle+\langle\bar{s}s\rangle}{12\pi^2f_\eta}+\frac{3f_{3\eta}}{4\sqrt{2}\pi^2}]$$

$$-\frac{4M^2}{3f_\eta}[(m_d\langle\bar{u}u\rangle + m_u\langle\bar{d}d\rangle)\langle\bar{s}s\rangle + m_s\langle\bar{u}u\rangle\langle\bar{d}d\rangle]$$

$$-\frac{m_\eta^2}{108f_\eta}[\langle\bar{u}u\rangle + \langle\bar{d}d\rangle + \langle\bar{s}s\rangle] < \frac{\alpha_s}{\pi}\mathcal{G}^2 >$$

$$+\frac{m_0^2}{18f_\eta}[-7(m_d\langle\bar{u}u\rangle + m_u\langle\bar{d}d\rangle)\langle\bar{s}s\rangle + 2m_s\langle\bar{u}u\rangle\langle\bar{d}d\rangle]. \quad (25)$$

Here we use $(2\pi)^4\lambda_\Lambda^2 = 1.64 GeV^6$. It is straightforward to show that starting from the Eq.(25) and applying Eq.(17) one returns to the Eq.(20).

4 Analysis of the sum rules and discussion

We have calculated numerically RHS's of the sum rules multiplied by exponential terms $e^{M^2/(m_\Sigma^2)}$ and $e^{(M^2/m_\Lambda^2)}$ (see Fig.1) and deduced coupling constants $g_{\eta\Lambda\Lambda} = -3.39$ and $g_{\eta\Sigma^0\Sigma^0} = 2.24$ in the borel parameter window $1.0 \le M^2 \le 2.0$ GeV^2 upon using the values of parameters cited in the text. We arrive at the parameters $g_{\eta\Lambda\Lambda}A_\Lambda = -1.92$ GeV^{-2} and $g_{\eta\Sigma\Sigma}A_\Sigma = 1.2$ GeV^{-2} in the chosen range of M^2. The main result is that the ratio of the discussed constants is practically stable in the borel parameter window $1.0 \le M^2 \le 2.0$ GeV^2 (see Fig.2) and equal to

$$\frac{g_{\eta\Sigma^0\Sigma^0}}{g_{\eta\Lambda\Lambda}} = -0.66.$$

Remind that in the strict linear $SU(3)$ scheme these constants are equal (± 12.8) up to a sign as seen from the Eq.(3). These values differ drastically from those given by $SU(3)$ scheme

$$\frac{g_{\eta\Lambda\Lambda}}{g_{\eta\Lambda\Lambda}|_{SU(3)}} = 0.26, \qquad \frac{g_{\eta\Sigma^0\Sigma^0}}{g_{\eta\Sigma^0\Sigma^0}|_{SU(3)}} = 0.18.$$

This result is in accord with that obtained in [11] for coupling constants $g_{\eta\Sigma\Sigma}$ and $g_{\eta\Xi\Xi}$.

New relations for strong coupling constants are derived in the $SU(3)$ and between QCD Borel sum rules for $\eta\Sigma\Sigma$ and $\eta\Lambda\Lambda$ couplings. It is shown that starting from the sum rule for the coupling constant $g_{\eta\Sigma\Sigma}$ it is straightforward to obtain the corresponding sum rule for the $g_{\eta\Lambda\Lambda}$ *et vice versa*. The derived sum rule is used to obtain the value of the $g_{\eta\Lambda\Lambda}$ coupling constant. Calculations show large $SU(3)$ symmetry breaking. Our result is the net D constant $SU(3)$ breaking as in the strict $SU(3)$ scheme both constants $g_{\eta\Lambda\Lambda}$ and $g_{\eta\Sigma\Sigma}$ are given by the D coupling and should be equal up to a sign.

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статья

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Валерий Семенович Замиралов

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